Symmetric User Grouping for Multicast and Broadcast Polling in IEEE 802.16 Networks

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Abstract

In this paper we address the bandwidth reservation in IEEE 802.16 standard via multicast and broadcast polling mechanisms. It is shown that symmetric user grouping with the same QoS requirements does not change the capacity of the random multiple access system. Binary exponential backoff protocol is then investigated as it is standardized for both multicast and broadcast polling. We establish that symmetric user grouping does not essentially increase the performance of this protocol.

I. Introduction

IEEE 802.16 standard [1] provides a high-speed channel access to various multimedia services. It specifies both physical (PHY) and media access control (MAC) layers with a special emphasis on the concept of quality of service (QoS).
From the MAC layer point of view IEEE 802.16 is a schedule-based system, which allows for several ways of bandwidth reservation. In [2] different reservation schemes are studied without any practical system considered.

Among others IEEE 802.16 defines a random access procedure of the bandwidth requesting and the binary exponential backoff (BEB) protocol is used as the means of the collision resolution. The asymptotic behavior of BEB protocol was extensively studied in the framework of two information theoretic models [3]. The infinite population model addresses the ultimate performance characteristics of the protocol, whereas in the framework of the finite population model the limits of the practical operation are established.

In order to evaluate the performance of the multicast and broadcast polling in IEEE 802.16 we begin with a short description of the standard in Section 2. The reference information theoretic model is described in detail in Section 3. Section 4 contains the general result on the infeasibility of symmetric user grouping for the infinite population model, while Section 5 addresses the behavior of BEB protocol for the finite population model. Finally, Conclusion summarizes the paper.

II. IEEE 802.16 brief description

The mandatory IEEE 802.16 operational mode assumes there is a base station (BS) that is connected to one or more subscriber stations (SSs). All the communication between the SSs is carried out via the BS. For the sake of brevity we refer to the SSs as to the system users in what follows.

Two separate communication channels are dedicated to the data packets exchange between the BS and the users. In the downlink channel the packets are directed to the users, while in the uplink channel they are directed to the BS. IEEE 802.16 defines two schemes of multiplexing the aforementioned channels. Time division duplex (TDD) assumes that the frame is separated into two parts in the time domain (see Fig. 1). Frequency division duplex suggests attaching the channels to the non-overlapping frequency bands.

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The downlink channel is broadcast in which the BS is the only station to send data packets. By contrast, the uplink channel is shared among all system users on the reservation basis. Simplifying, once a user acquires a pending data packet it first participates in the bandwidth reservation stage to obtain a part of the uplink channel resources. After the resource has been granted to a user, it proceeds with sending its data packet(s).

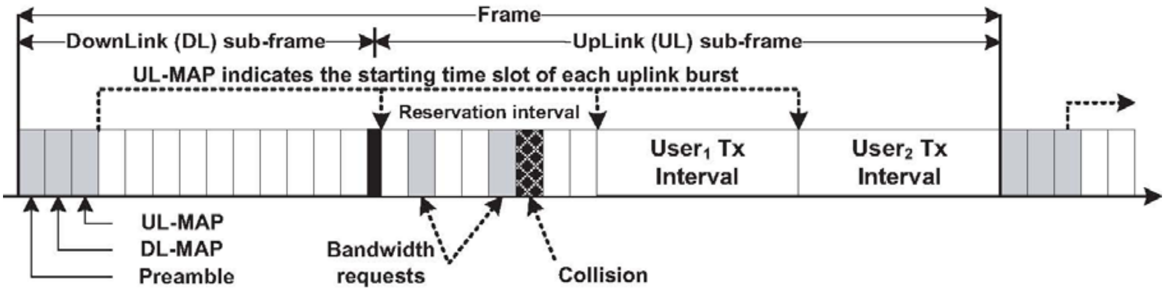

Fig. 1. TDD frame structure.

IEEE 802.16 defines unicast, multicast and broadcast polling mechanisms at the bandwidth reservation stage. Unicast polling assumes no contention between users and each user is eventually granted a slot to send its bandwidth request. As this scheme offers high overhead on the system performance, it is practically inapplicable when the number of system users is sufficiently high. Broadcast polling offers several contention slots per frame and each user chooses randomly in which to send its request. In case of a collision a user retransmits the request following the rules of the BEB protocol. Multicast polling allows for splitting users into the integer number of groups and the users of a particular group are restricted to use only the dedicated part of the contention slots.

III. System model

We proceed with the formulation of the reference information theoretic model following the approach of [4] and [5]. The system time is divided into adjacent frames of equal duration. The frames are enumerated with integer and nonnegative numbers. Suppose there are M users in the system. We formulate additional assumptions about the way the requests arrive into the system and are transmitted.

Assumption 1. According to IEEE 802.16 standard each user may potentially establish multiple connections with the BS using different negotiated QoS parameters, and a bandwidth request can be issued on a per-connection or a per-station basis. In what follows we assume that each user has only one connection at a time and all the connections belong to the same QoS class.

Assumption 2. Each frame comprises K equal contention slots for the request transmissions. K is constant throughout the system operation.

Assumption 3. In each slot one of the following situations may occur:

− exactly one user transmits its request (success);

− none of the users transmit the request (empty);

− two or more users transmit their requests simultaneously, which results in the corruption of all the requests at the BS (collision).

Assumption 4. The uplink channel is noise-free. Therefore, the BS faultlessly determines which situation occurred in a slot. If only one user transmits, then the BS always decodes the bandwidth request successfully.

Assumption 5. For each arrived data packet a separate bandwidth request is generated. As we concentrate on the bandwidth reservation process, we assume the virtual input flow of requests into the system.

Assumption 6. By monitoring user activity in the frame $t-1$ the BS makes a schedule for the uplink sub-frame of the frame t and broadcasts this schedule in the downlink sub-frame of the frame

t. A user receives the feedback from the request transmission in the frame $t-1$ by the beginning of the frame t.

According to the standard this is not the case. Feedback information is not explicitly transmitted to a user. A special request timeout is used to wait for the uplink grant from the BS, and only if it is expired, the request transmission is considered corrupted. We make this 'immediate' feedback assumption for the simplicity of the analysis only. All the forthcoming derivations may be generalized for the case of the 'delayed' feedback.

Assumption 7. The downlink channel is noise-free. Therefore, a user faultlessly receives the schedule and the request transmission feedback from the BS.

Assumption 8. Denote the random number of the new request arrivals to the user i in the frame t by $X_i^{(t)}$. For all $t \geq 0$ and $i = 1,...,M$ the random variables $X_i^{(t)}$ are independent and identically distributed (i.i.d.). Assume also that at most one new request arrives to a user per frame with the probability y. Thus, $E[X_i^{(t)}] = y$ for all $t \geq 0$ and $i = 1,...,M$ as well as $E[\sum_{i=1}^{M} X_i^{(t)}] = My = \Lambda$. The value of Λ is hereinafter referred to as the cumulative input rate and the considered input flow constitutes a Bernoulli flow.

Parameter	Description
M	User population
N	Size of each group
$G = \frac{M}{N}$	Number of groups
K	Total number of slots per frame
L	Number of slots per frame per group
Q	Maximum number of retransmission attempts
W	Initial contention window
m	Maximum backoff stage

Table 1. System model parameters.

IV. Infinite population model

Following the approach of [3] we allow the number of users in the system M to increase infinitely and the probability of a request arrival y to decrease simultaneously so that their product remains constant, that is $My = const = \Lambda$. Then the limit of the cumulative arrival process given by **Assumption 8** is Poisson, i.e. $\lim_{M\to\infty} \Pr\{\sum_{j=1}^{M} X_j^{(t)} = i\} = \frac{\Lambda^i}{i!} e^{-\Lambda}$. Below we make the basic definitions and introduce lossy and lossless system types as follows.

A. Lossy system

Definition 1. The algorithm A from the class of algorithms for the lossy system $A \in \mathrm{A}_{lossy}$ is defined as a rule that allows a user with a pending request to determine whether it should transmit this request in the following slot s or discard it. If a request is discarded then the corresponding data packet is lost [6].

Definition 2. We introduce a random variable $Z^{(t)}$, which is the number of the successful request transmissions in a frame comprising K slots. Clearly, $Z^{(t)} \in \{0,1,...,K\}$. Define the random variable $\Psi_A(K,\Lambda,s)=\frac{\sum_{j=0}^{s}Z^{(t)}}{sK}$. The limit of this expression for s, if it exists, represents the output rate per slot of the algorithm A in the lossy system, that is $\Psi_A(K,\Lambda)=\lim_{s\to\infty}\Psi_A(K,\Lambda,s)$.

Definition 3. The throughput of the algorithm A in the lossy system is the maximum achievable output rate for all the input rates, which implies:

$$T_A(K)=\sup_{\Lambda}\Psi_A(K,\Lambda). \tag{1}$$

Definition 4. The capacity of the lossy system is the maximum throughput over the class $\mathrm{A}_{lossy}(K)$ of the algorithms with K slots per frame:

$$C_{lossy}(K)=\sup_{A\in \mathrm{A}_{lossy}(K)}T_A(K). \tag{2}$$

Notice, that the throughput value characterizes the behavior of an algorithm, whereas the capacity gives the ultimate performance threshold for the entire lossy system.

B. Lossless system

Definition 5. The algorithm A from the class of algorithms for the lossless system $A\in \mathrm{A}_{lossless}$ is defined as a rule that allows a user with a pending request to determine whether it should transmit this request in the following slot s. Notice, that no discard rule is specified and, consequently, requests are never lost.

Definition 6. The request delay for an algorithm is the time interval from the moment of the request generation to the moment of its successful transmission. The delay $\delta_A(K,\Lambda)$ is a random variable. We inject a new request into the system at the randomly chosen slot s, and denote the delay of this request as $\delta_A^{(s)}(K,\Lambda)$.

Definition 7. The mean delay (referred to as virtual mean delay in [7]) is defined as:

$$D_A(K,\Lambda)=\overline{\lim_{s\to\infty}}E[\delta_A^{(s)}(K,\Lambda)]. \tag{3}$$

Definition 8. The transmission rate (tenacity) of the algorithm A in the lossless system is the maximum input rate that can be sustained by the algorithm with finite request delay:

$$R_A(K)=\sup_{\Lambda}\{\Lambda : D_A(K,\Lambda)<\infty\}. \tag{4}$$

Definition 9. The capacity of the lossless system is the maximum possible rate over the class $\mathrm{A}_{lossless}(K)$ of the algorithms with K slots per frame:

$$C_{lossless}(K)=\sup_{A\in \mathrm{A}_{lossless}(K)}R_A(K). \tag{5}$$

The exact value of the capacity is not yet established. However, the best known upper bound on the capacity $C_{lossless}(1)$ was established in [8] and is shown to be $\overline{C}_{lossless}(1)=0.587$. The best known part-and-try algorithm was proposed in [9] and its rate is $R_{pt}=0.487$. In subsequent years it was slightly improved, but the core idea of the algorithm remained unchanged.

Notice again, that the rate value characterizes the behavior of an algorithm, whereas the capacity gives the ultimate performance threshold for the entire lossless system.

C. Symmetric grouping

Here we concentrate on showing that the symmetric grouping of users (the one with the same QoS requirements) does not increase the ultimate measure of the system performance, namely, its capacity. The below arguments may be repeated similarly for both lossy and lossless types of the system. Below we demonstrate the proof for the lossless system, but omit the lower 'lossless' index at A and C as redundant.

1. Firstly, consider the system without the framing structure. The system time is divided into equal slots and a user is restricted to start its request transmission in the beginning of a slot. The algorithm A in this system $\mathrm{A}_{slotted}$ may again be defined as a rule that allows a user with a pending request to determine whether it should transmit this request in the following slot s. The feedback of the user transmission is available by the beginning of the next slot $s+1$.
2. Now we additionally divide the system time into frames with each frame comprising some integer and constant number of slots K. However, the feedback is still available after each slot. It is assumed that all the system users monitor the system activity from the start of its operation. Therefore, all the users determine the situation in each slot similarly and the introduction of frames neither improves nor degrades the system performance. The conclusion we draw from this fact is that the set of all the algorithms for this system A_{framed} coincides with the set of algorithms for the slotted system, that is $\mathrm{A}_{framed} = \mathrm{A}_{slotted} = \mathrm{A}(1)$. Analogously to the **Definition 9** we define the capacity of the framed system as $C(1) = \sup_{A \in \mathrm{A}(1)} R_A(1)$.
3. We change the feedback availability for the framed system and let a user know the consequences of a request transmission only in the beginning of the next frame, that is, once in K slots. An alternative system with 'delayed' feedback was considered in [10]. We define the algorithm A for this system $\mathrm{A}(K)$ as before and conclude that with the restriction on the feedback availability the set of all possible algorithms is narrowed in comparison to the respective set for the framed system, which yields $\mathrm{A}(K) \subset \mathrm{A}(1)$. From the above and the two definitions of capacity $C(1)$ and $C(K)$ (5) it immediately follows that $C(K) \le C(1)$.
4. To any algorithm A from the set $\mathrm{A}(1)$ an algorithm A^* may be put into correspondence that belongs to $\mathrm{A}(K)$, such as $R_{A^*} = R_A$. For this it is sufficient to split all the users of the framed system into K equal groups and restrict the slots available for each group to one slot per frame. For instance, group number one monitors and transmits in the first slot of each frame, group number two – in the second, etc. Therefore, for each group the feedback is available at the beginning of the next slot, dedicated to this particular group, which corresponds to the slotted system.
5. From the definition of the capacity and the above (see 3, 4) it follows that $C(1) = C(K)$, that is, the capacity does not change for the framed system. Moreover, when all the system users are already split into equal groups with L slots for each of them, the capacity does not change either, i.e. $C(1) = C(L) = C(K)$. We conclude that symmetric grouping of users leaves the system capacity unchanged.

V. Finite population model

Below we investigate the finite population model with M users. The contention resolution process for multicast and broadcast polling is controlled by the BEB protocol. We, therefore, concentrate on the analysis of this protocol for both lossy and lossless systems.

A. BEB protocol operation

Rule 1.1. If a new bandwidth request arrives to a user in the frame $t-1$ and this user has no other pending requests, it transmits the request in the frame t (transmission attempt). The slot for the request transmission is sampled uniformly from the number of contention slots dedicated to the group the user belongs to. Notice, that in case of broadcast polling the user may choose between all the contention slots K of the frame t, whereas in case of multicast polling the choice is narrowed to L slots of the respective multicast group.

Rule 1.2. If a request is ready for retransmission at the beginning of the frame t at its i-th retransmission attempt ($i > 0$), a user chooses a number (backoff counter) in the range $\{0,1,...,2^{\min(m,i)}W-1\}$ uniformly, where W and m are the parameters of the BEB protocol, named initial contention window and maximum backoff stage respectively and i is the number of collisions this request suffered from so far. The user then defers the request retransmission for the chosen number of slots, accounting only for the slots dedicated to its group.

Rule 2.1. If, after receiving the feedback from the BS, the user determines that its last request collided, it increments the collision counter i for this request. If this counter coincides with the maximum allowable number of retransmission attempts Q, then the request together with the corresponding data packet is discarded and the collision counter is reset to $i = 0$.

Rule 2.2. If, after receiving the feedback from the BS, the user determines that the (re)transmission of the last request was successful, it resets the collision counter to $i = 0$.

B. Lossy system

In what follows we address the throughput of the BEB protocol T_{BEB} in the lossy system for the minimum packet delay case. To achieve this like in [11] we set the number of retransmission attempts $Q = 0$ and denote the respective BEB throughput value for a single transmission attempt by T_{BEB}^1.

IEEE 802.16 neither offers guidelines for selecting W, m and K nor it defines any grouping rules. To avoid wasting multicast/broadcast polling slots we set $W = \frac{lK}{G} = lL$, where l – natural number and $l \geq 1$. In the considered case of $Q = 0$ we immediately obtain $l = 1$ and $m = 0$.

In order to derive the sought throughput value T_{BEB}^1 for broadcast ($G = 1$) and multicast ($G > 1$) polling we address a technique similar to that in [12]. In each slot at most one request may be transmitted. We introduce a random variable $Z^{(i)}$ that is equal to 1 in case of success in the slot i and is equal to 0 otherwise. Notice, that as the number of users in each group is constant and the users are independent, it is sufficient to obtain the expectation of the sum $Z^{(i)}$ over L for one group only. Clearly, this expected value gives T_{BEB}^1 value, that is:

$$T_{BEB}^1 = \frac{E[\sum_{i=1}^{L} Z^{(i)}]}{L} = E[Z^{(i)}]. \tag{6}$$

The expected value of $Z^{(i)}$ represents the probability of a success in a slot, which happens iff one of N users in a group chooses this slot for its request transmission, yielding:

$$T_{BEB}^1 = E[Z^{(i)}] = \Pr\{Z^{(i)} = 1\} = \frac{yN}{L}(1 - \frac{y}{L})^{N-1}, \tag{7}$$

where the value of y represents the probability of a request arrival to a user in a frame according to the Bernoulli input flow (see **Assumption 8**).

By calculating the first derivative of (7) for y and imposing it equal to 0, we establish the 'optimal' value of the input rate y_0 that results in the maximum throughput value as:

$$y_0 = \frac{L}{N}. \tag{8}$$

Fig. 2 demonstrates the function T_{BEB}^1 for different number of groups G, $K = 8$ and $M = 40$. We observe that multicast polling outperforms broadcast polling for small input rates y, whereas the situation reverses for moderate and high input rates.

We also notice that the gap between the cases with $G = 1$ and $G = 8$ is the most significant and shows the maximum possible gain/loss from the use of either of polling techniques. We plot the dependence of this maximum gain/loss on the input rate in Fig. 3. The demonstrated approach allows the derivation of a closed-form expression for the maximum broadcast polling gain/loss function as follows:

$$f(y) = \frac{yM}{K}(1-\frac{y}{K})^{M-1} - \frac{yN}{L}(1-\frac{y}{L})^{N-1}. \quad (9)$$

We conclude that despite the fact that the use of multicast or broadcast polling demonstrates a throughput trade-off for different values of request arrival rate, the maximum possible gain/loss is negligible in comparison to the achievable throughput. Therefore, it is not reasonable to split users into multicast groups for the considered minimum delay case ($Q = 0$), as the gain is minor, but IEEE 802.16 overhead increases as the number of groups grows [1].

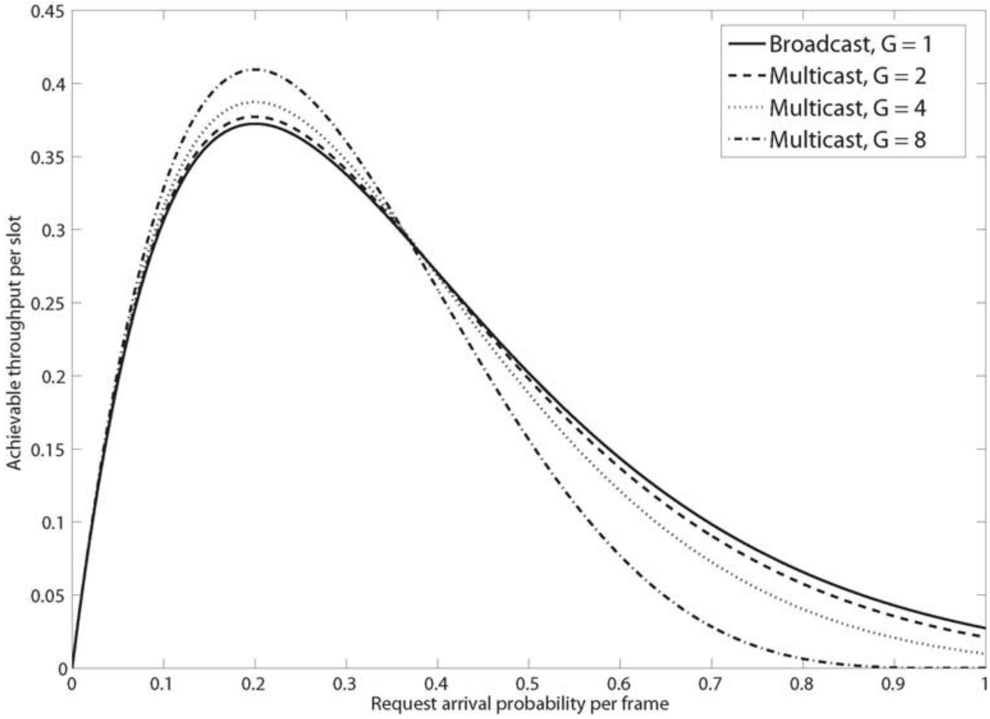

Fig. 2. BEB throughput for multicast and broadcast polling.

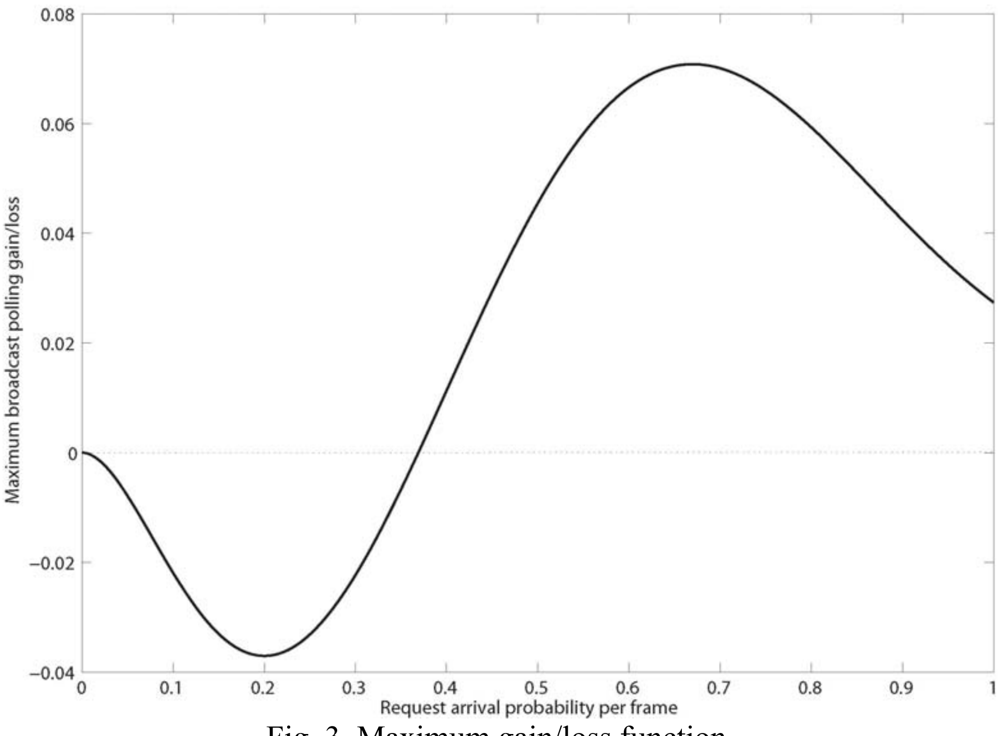

Fig. 3. Maximum gain/loss function.

C. Lossless system

Below we concentrate on the analysis of finite population lossless system for which we set the number of request retransmissions Q infinite. In this case the performance measure of the BEB protocol is its rate R_{BEB}.

We introduce the stochastic process $c(s)$ that represents the value of the randomly sampled backoff counter at time s given that the number of collisions suffered by a request so far is $b(s)$. A discrete and integer time scale is also adopted, where s and $s+1$ correspond to the start times of two successive slots. We demonstrate our approach for broadcast polling as the example. All the below derivations may be generalized for the case of multicast polling with N users per group.

We notice, that according to the BEB rules a user after its (re)transmission attempt does not start the backoff process immediately, but rather waits for the beginning of the next frame. Assume, that the (re)transmission attempt occurs in slot s in the frame that consists of K slots. Therefore, the user waits $K-s$ slots before resuming the backoff procedure. At its every retransmission attempt a user may be regarded as choosing the frame to retransmit in first and then choosing one of K slots in this frame. Thus, the number of slots before the (re)transmission in a frame is sampled uniformly in the range $[0,...,K-1]$. Denote the waiting time counter as $a(s)$, which accounts for the slots after the (re)transmission attempt by a user and before the start of the next frame.

The considered stochastic process represents a Markov chain analogous to one described in [13] and [14], but with the addition of $K-1$ idle states, which correspond to the possible waiting time counter values. The transition probabilities for these additional states may be computed as follows:

$$\begin{aligned} &\Pr\{a(s+1)=k-1 \mid a(s)=k\}=1, k=1,...,K-1, \\ &\Pr\{a(s+1)=k \mid b(s)=0\}=\frac{1}{K}, k=1,...,K-1. \end{aligned} \tag{10}$$

Let $b_{i,j}=\lim_{s\to\infty}\Pr\{b(s)=j,c(s)=i\}$, $a_k=\lim_{s\to\infty}\Pr\{a(s)=k\}$, where $i=\{0,...,m\}$, $j=\{0,...,2^iW-1\}$ and $k=\{1,...,K-1\}$ be the stationary distribution of the considered Markov chain. As the probability of a (re)transmission attempt in a slot is equal to $\sum_{i=0}^{m} b_{i,0}$, we establish:

$$\begin{aligned} &a_k=\frac{k}{K-k}\sum_{i=0}^{m} b_{i,0} \Rightarrow \\ &\sum_{k=1}^{K-1} a_k=\frac{K-1}{2}\sum_{i=0}^{m} b_{i,0}=\frac{K-1}{2}\cdot\frac{b_{0,0}}{1-p_c}, \end{aligned} \tag{11}$$

where p_c is the conditional collision probability, which is equal to the probability that at least one of the remaining $M-1$ users (re)transmits:

$$p_c=1-(1-p_t)^{M-1}. \tag{12}$$

Accounting for the normalization condition:

$$1=\sum_{i=0}^{m}\sum_{j=1}^{2^iW} b_{i,j}+\sum_{k=1}^{K} a_k, \tag{13}$$

we notice that the first term is given in [13]. Summarizing, the probability p_t that a user (re)transmits in a randomly chosen slot is readily obtained as:

$$p_t=\sum_{i=0}^{m} b_{i,0}=\frac{2(1-2p_c)}{(1-2p_c)(W+K)+p_cW(1-(2p_c)^m)}. \tag{14}$$

Equations (12) and (14) represent a nonlinear system with two unknowns p_c and p_t, which may be solved numerically. The resulting R_{BEB} value is finally given by the probability of one (re)transmission in a slot:

$$R_{BEB} = Mp_t(1-p_t)^{M-1}. \quad (15)$$

The above approach allows the derivation of the optimal (re)transmission probability value that gives the maximum BEB protocol rate over all possible pairs of (W, m). It may be shown that this maximum value is reached for $m = 0$. Below we consider the optimal system in more detail.

Substituting $m = 0$ into (14) we obtain that $p_t = \frac{2}{W_0 + K}$, where W_0 is the optimal initial contention window value. Notice that (15) closely resembles the expression (7), which is maximized for $\frac{yN}{L} = 1$. Therefore, the expression (15) itself is maximized for $Mp_t = \frac{2M}{W_0 + K} = 1$. Finally, W_0 is obtained as $2M - K$, or, accounting for the possible grouping of users:

$$W_0 = 2N - L. \quad (16)$$

It should be emphasized, that the rate of the optimized BEB protocol with $m = 0$ and W_0 gives precisely the same value as calculated by (7) for the lossy system. However, the usage of the optimal initial contention window W_0 in IEEE 802.16 standard is not straightforward, as it may not be an integer power of two. For this reason we depict the BEB rate for various values of m and different initial contention windows in Fig. 4. We see, that for the example system with $M = 40$, $K = 8$ and broadcast polling, $W_0 = 72$. The BEB rate given by $W = 32$ and $m = 2$ is almost as high as the optimal one. Summarizing, our approach allows the optimization of BEB parameters in terms of the highest achievable rate.

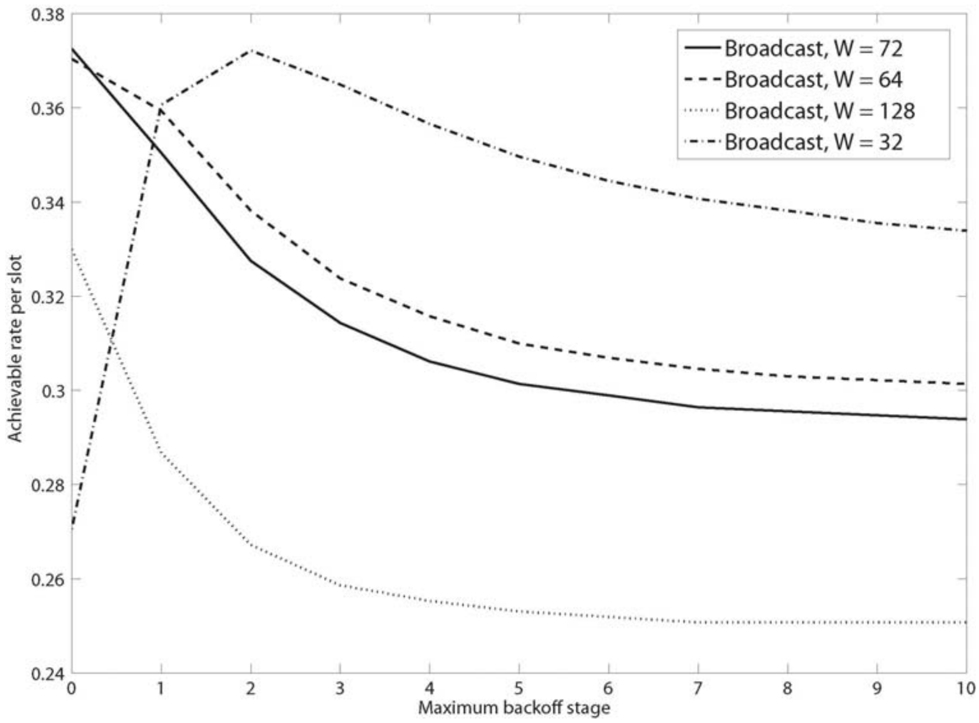

Fig. 4. BEB rate for broadcast polling.

VI. Conclusion

Below we start with some simulation results that are used to make final conclusions on broadcast and multicast polling efficiency. In Fig. 5 we demonstrate the throughput of the system, where the maximum number of retransmission attempts is set to some natural number, that is, $Q \geq 1$. Therefore, this system represents the intermediate case between those discussed in Section 5.2 and Section 5.3.

We see that for all the values of Q the throughput converges to the value indicated by (15) and (7). However, the convergence is faster for the greater Q value as less requests get discarded. An important conclusion from this is that regardless of the considered system (lossy or lossless) the performance measure of the BEB protocol is unchangeable, i.e. $T^{1}_{BEB} = T^{Q+1}_{BEB} = T_{BEB}$ and $T_{BEB} = R_{BEB}$.

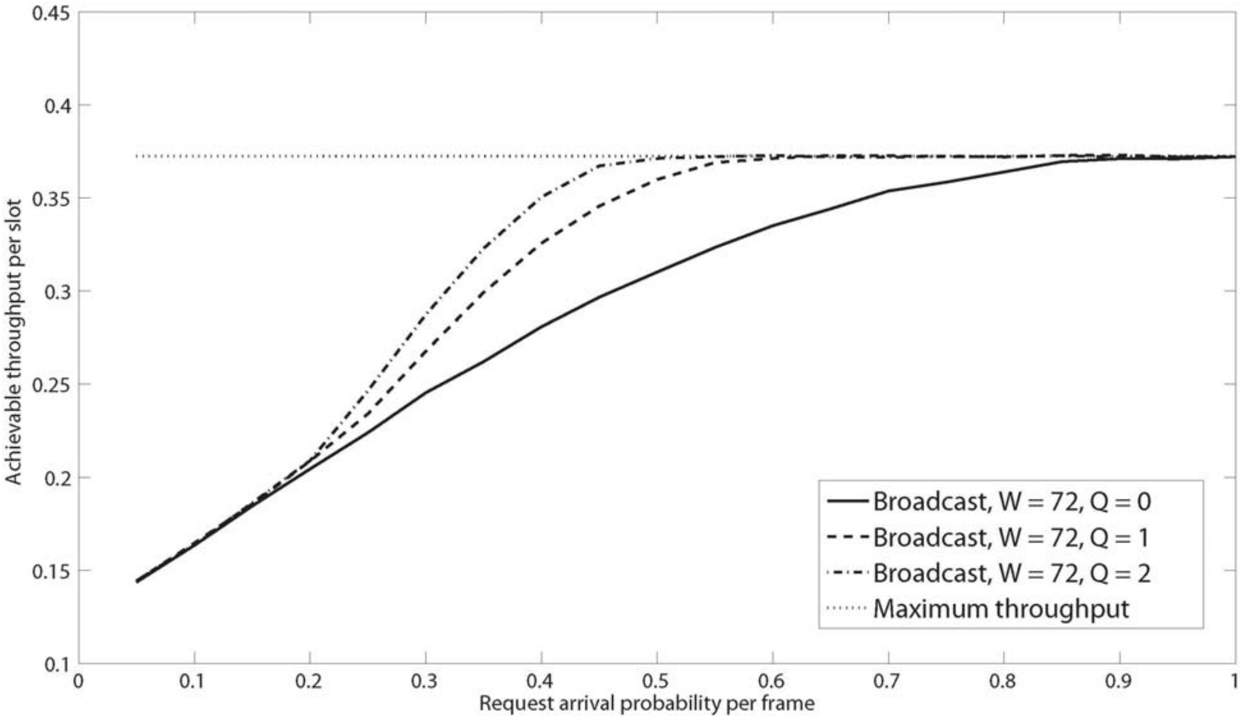

Fig. 5. BEB simulation throughput for broadcast polling.

Summarizing, we studied the theoretical infinite population model and showed that symmetric user grouping does not change its ultimate performance measure (capacity). We have formulated two system types: lossy and lossless and addressed the performance of BEB protocol for these system types. It was shown that the BEB rate is precisely the same as its throughput, that is, the optimal behavior of BEB is independent of the system type.

Although multicast and broadcast polling indeed demonstrate a performance trade-off for symmetric user grouping, this gain was shown to be negligible for any practical scenarios. Subject to proper IEEE 802.16 standard amendments, another (asymmetric) grouping may be proposed which also could be evaluated with the demonstrated approaches.

References

[1] IEEE Std 802.16e-2005, Piscataway, NJ, USA, December 2005.

[2] *I. Rubin.* Access-control disciplines for multi-access communication channels: Reservation and TDMA schemes. IEEE Transactions on Information Theory, 25(5):516–536, 1979.

[3] *M. Paterakis, L. Georgiadis, and P. Papantoni-Kazakos.* On the relation between the finite and the infinite population models for a class of RAA's. IEEE Transactions on Communications, 35:1239–1240, 1987.

[4] *B. Tsybakov and V. Mikhailov.* Free synchronous packet access in a broadcast channel with feedback. Problems of Information Transmission, 14(4):259–280, 1978.

[5] *D. Bertsekas and R. Gallager.* Data Networks. Prentice Hall, 1992.

[6] *B. Tsybakov.* One stochastic process and its application to multiple access in supercritical region. IEEE Transactions on Information Theory, 47(4):1561–1569, 2001.

[7] *B. Tsybakov.* Survey of USSR contributions to random multiple-access communications. IEEE Transactions on Information Theory, 31(2):143–165, 1985.

[8] *B. Tsybakov and N. Likhanov.* Upper bound on the capacity of a random multiple access system. Problems of Information Transmission, 23(3):224–236, 1987.

[9] *B. Tsybakov and V. Mikhailov.* Random multiple packet access: Part-and-try algorithm. Problems of Information Transmission, 16(4):305–317, 1980.

[10] *B. Tsybakov and M. Berkovskii.* Multiple access with reservation. Problems of Information Transmission, 16(1):35–54, 1980.

[11] *O. Alanen.* Multicast polling and efficient VoIP connections in IEEE 802.16 networks. 10th ACM Symposium on Modeling, analysis, and simulation of wireless and mobile systems, 1:289–295, 2007.

[12] *N. Abramson.* The throughput of packet broadcasting channels. IEEE Transactions on Communications, 25(1):117–128, 1977.

[13] *G. Bianchi.* Performance analysis of the IEEE 802.11 distributed coordination function. IEEE Journal on Selected Areas in Communications, 18(3):535–547, 2000.

[14] *A. Vinel, Y. Zhang, M. Lott, and A. Tiurlikov.* Performance analysis of the random access in IEEE 802.16. IEEE 16th International Symposium on Personal, Indoor and Mobile Radio Communications, 3:1596–1600, 2005.