Practical Traffic Generation Model for Wireless Networks

Sergey Andreev¹, Alexey Anisimov², Yevgeni Koucheryavy¹, and Andrey Turlikov³

Tampere University of Technology¹ (TUT), FINLAND sergey.andreev@tut.fi yk@cs.tut.fi H&NM Motorola Software Organization², RUSSIA alexey.anisimov@motorola.com State University of Aerospace Instrumentation³ (SUAI), RUSSIA turlikov@vu.spb.ru

Abstract. In this paper we address traffic engineering problems for wireless communications by proposing a practical technique to construct a DBMAP-based model equivalent to an IPP-based model. On the one hand, IPP-based (ON-OFF) models are widespread in practice to simulate the application layer traffic in wireless networks. On the other hand, DBMAP-based models are backed by the developed matrixanalytic methods. The derived model is easy to use and allows to apply matrix-analytic methods in the performance evaluation of the emerging networking technologies.

Keywords: DBMAP, IPP, traffic engineering, wireless networks

1 Introduction and Background

Recent advances in wireless communications and the proliferation of networking technologies dictate the need for their deeper and more accurate performance evaluation. Practically, both emerging and existing telecommunication services, including voice, video and data, demand the respective quality of service (QoS) guarantees. To ensure that the adequate QoS is reached it is often required to analyze and simulate the considered communication system. Networking standards traditionally define the physical (PHY) and the Media Access Control (MAC) system layers, but leave out of scope the properties of the upper layers. Therefore, the choice of an adequate traffic model to evaluate the QoS performance of a wireless network is a challenging task [1].

Many research works address the derivation of an appropriate traffic model. In particular, empirical models to mimic the behavior of particular upper layer protocols are known. In [2] a statistics-based model of the widespread Hypertext Transfer Protocol (HTTP) is constructed. It can be used to reproduce the corresponding network traffic. Another example of more sophisticated wireless environment is given by [3]. The model enables user mobility, which increases its practical relevance. Peer-to-peer traffic model is discussed in [4], where the realistic packet traces have been analyzed. However, these models strongly depend on the initial statistics and the selected simulation scenario and could not serve as a generic simulation methodology.

The development of more versatile traffic models that could be used in a multitude of MAC/PHY simulation scenarios has been pursued by [5] as an extension of the author's earlier document [6]. The models adopt the Interrupted Poisson Process (IPP) and the superposition of several IPPs to produce typical Ethernet and Internet traffic over a wireless network. The resulting ON-OFF model is scalable, easy to use and is reported to provide accurate results. For the above reasons the IPP-based ON-OFF model was proposed for the prominent IEEE 802.16 [7] networking standard. This model has later been enhanced in [8] for the future version of the standard, which is about to be finalized. The use of the ON-OFF model for the performance evaluation of the IEEE 802.16 sleep mode mechanism was demonstrated by [9] and also for the end-to-end delay evaluation of IEEE 802.11 standard by [10].

The wide recognition of the ON-OFF traffic models is confirmed by both their acceptance as a part of the most recent IEEE 802.16 evaluation methodology [11] and long-run WiMAX Forum approval [12]. These models are also used by the 3rd Generation Partnership Project 2 (3GPP2) [13]. All the above makes their necessity for wireless system simulation doubtless. The IPP-based ON-OFF models form an important group of traffic models according to the classifications in [14], [15] and [16], being a useful special case of more general Markov Modulated Poisson Process (MMPP) models. In turn, MMPP is a subclass of Batch Markovian Arrival Process (BMAP), which has been extensively analyzed and thus provides a more comprehensive traffic model, while still preserves the analytical tractability [17].

The BMAP was first introduced in [18] and later used by [19] and [17] to model the IP network traffic with variable packet length. In [20] a more transparent and consistent notation was proposed and BMAP was renamed to become the Discrete-time Batch Markovian Arrival Process (DBMAP). The use of DBMAP for the joint consideration of IP packet length and packet arrival times was addressed by [21]. Many well-known arrival processes are shown to be the special cases of the DBMAP, including the Bernoulli arrival process, the Markov Modulated Bernoulli Process (MMBP), the batch Bernoulli process with correlated batch arrivals and others [22]. Layered video traffic, such as MPEG-4, could be modeled by DBMAP as shown in [23], [24] and [25]. A DBMAP model of H.264/SVC scalable video is constructed in [26].

The applicability of the DBMAP-based models, however, is not limited to the IP traffic and video applications. In [27] and [28] useful analytical models of IEEE 802.16 sleep mode mechanism operation with DBMAP arrivals were proposed. The MAC layer collision resolution algorithms with DBMAP input traffic were investigated in [29], [30] and [31]. Additionally, some papers, e.g. [32], [33], [34] and [35], analyze DBMAP variations, as well as Markovian Arrival Processes (MAPs) in the framework of the matrix-analytic theory.

Unfortunately, the aforementioned analytical results so far have limited value for the real-world communication systems due to the gap between the complicated arrival processes under consideration and the simpler models, that are proposed for the performance evaluation of the actual networking standards by [11], [12] and [13]. In this paper we fill in this gap by proposing a simple technique to construct a DBMAP that closely corresponds to a practical IPPbased ON-OFF model. Therefore, the existing matrix-analytic results could be applied directly to evaluate the QoS performance of the practical wireless communication systems, such as IEEE 802.16, IEEE 802.11 and 3GPP Long Term Evolution (LTE).

The rest of the paper is structured as follows. Section 2 describes known traffic models for wireless communication networks and shows different approaches to traffic modeling. In Section 3 we describe an important class of the DBMAP processes, for which strong analytical results are known. Section 4 demonstrates a simple approach to construct an equivalent DBMAP-based model for the wellknown realistic IPP-based model. Finally, Section 5 establishes the similar performance of both existing and constructed models and Section 6 concludes the paper.

2 Traffic Models Description

As discussed in Section 1, the construction of an adequate traffic model is an important task in the performance evaluation of wireless communication networks. The behavior of the developed model should mimic the behavior of the realistic networking traffic.

We consider the HTTP traffic model recommended by [11], [12] and [13]. An example of the packet distribution during an HTTP session is presented in Fig. 1. The typical session is observed as a sequence of distinct ON and OFF periods during which the traffic is produced and not produced, respectively.



Fig. 1. Typical HTTP session structure

The durations of ON and OFF periods are distributed exponentially in a way that the mean duration of the ON period is $1/C_1$ and the mean duration of the OFF period is $1/C_2$. The probability that the model is in the ON state is

$$P_{ON} = \frac{C_2}{C_1 + C_2}.$$
 (1)

Similarly, the probability that the model is in the OFF state is

$$P_{OFF} = \frac{C_1}{C_1 + C_2}.$$
 (2)

As mentioned above, the ON-OFF traffic model is also known as the IPP [5]. The latter is a Markov chain with two states as depicted in Fig. 2. The chain is fully described by three parameters: the transition probability rate from the OFF state to the ON state, the packet arrival rate in the ON state and the transition probability rate from the ON state to the OFF state. The transition probability rate is defined as the number of transitions from a state to another per a unit of time.



Fig. 2. Generic ON-OFF model

According to Fig. 2 the value of C_1 is the transition probability rate between the ON and OFF states, C_2 is the similar transition probability rate in the opposite direction. Being in the ON state the model generates data packets according to the Poisson distribution with the arrival rate of λ_{ON} . Therefore, the packet inter-arrival time is distributed exponentially according to the distribution function

$$F(t) = 1 - e^{-\lambda_{ON} \cdot t}.$$
(3)

More advanced traffic models include the superposition of several IPPs, where each IPP has different parameters. The conventional notation here is to add the number of processes in the superposition before the name of the model. For instance, the 4IPP model corresponds to the combination of four interrupted Poisson processes. In what follows we concentrate on the case of the 1IPP traffic model for the sake of simplicity, which corresponds to the individual subscriber Internet scenario. The below table lists the parameters of such a model according to [5].

The parameters given in the table are normalized and should be scaled for a particular traffic arrival rate. For example, consider the target simulated traffic arrival rate of 50 Kbps. According to the previous research by Telcordia and Lawrence Berkeley Labs the average packet length is 192 bytes or 1536 bits [5]. Therefore, the number of packets per second equals to 50000/1536 = 32.552. Next we obtain the number of time_units in a second as 32.552/0.7278 = 44.7266

IPP model parameters	
Parameter	Value
Arrival rate in the ON state λ_{ON} , packets/time_unit	1.698
C_1 , transitions/time_unit	$1.445 \cdot 10^{-2}$
C_2 , transitions/time_unit	$1.084 \cdot 10^{-2}$
Averaged arrival rate λ , packets/time_unit	0.7278

time_units/sec. Finally, to establish the sought parameters of the traffic model that gives the arrival rate of 50 Kbps one should multiply all the values in the above table by the scaling factor of 44.7266.

The considered IPP model could be further generalized to a MMPP model subject to the higher number of states and different arrival rates corresponding to a state [14].

3 DBMAP Description

In this section we formally introduce the DBMAP and its main parameters. Consider the time axis, which is broken into equal time intervals called slots. We enumerate the slots with natural numbers and the slot number t corresponds to the time interval [t-1,t). For simplicity we refer to the slot number t as to the slot t in what follows. Consider also a stochastic process with the discrete state space, which we denote as **S**. Each slot t is put into one-to-one correspondence with a state from the considered state space: $S^t \in \mathbf{S}$. On the other hand, each slot t is put into one-to-one correspondence with an integer non-negative number X^t , which is the number of new packet arrivals during the slot t. We note that subject to such a discretization the arrival times of the X^t packets could not be distinguished.

Denote the probability of n new arrivals during the slot t and the transition of the considered stochastic process from state i to state j in the end of the slot t as follows:

$$\Pr\{X^t = n, S^{t+1} = j | S^t = i\} \triangleq b_{ij}(n). \tag{4}$$

For each value of n a corresponding matrix $\mathbf{B_n} = \{b_{ij}(n)\}$ could be derived. In the most general case the number of new arrivals n during a slot is unbounded. This corresponds to the case of infinitely-many matrices $\mathbf{B_n}$. The described process $\{X^t, S^t\}$ is named DBMAP in [20]. This process is fully described by a set of matrices $\mathbf{B_n}$ for all possible values of n. Below we introduce the main DBMAP-related definitions.

Consider a matrix **B**, which is obtained as the sum of matrices $\mathbf{B}_{\mathbf{n}}$ composed of the elements (4) for all possible values of n:

$$\mathbf{B} \triangleq \sum_{n=0}^{\infty} \mathbf{B}_{\mathbf{n}}.$$
 (5)

Then each element of the matrix **B** is the probability of a transition by the process $\{S^t\}$ from the state *i* to the state *j* in the end of the slot *t*:

$$b_{ij} = \Pr\{S^{t+1} = j | S^t = i\}.$$
(6)

We assume that the matrix \mathbf{B} composed of the elements (6) is aperiodic and irreducible. Such matrices are often referred to as primitive [30]. Similarly, a DBMAP process that corresponds to a primitive matrix \mathbf{B} is called primitive DBMAP. For the sake of simplicity we restrict our explorations to the case of the primitive DBMAPs.

We introduce the important characteristics of the primitive DBMAPs. Consider the mean number of new packet arrivals in the state i during a slot:

$$\lambda_i \triangleq \sum_n n \sum_j b_{ij}(n). \tag{7}$$

Accounting for the fact that the process $\{S^t\}$ is ergodic, we introduce the stationary probability p_i of finding the process $\{S^t\}$ in the state *i* conditioning on the fact that in the initial slot the process started from the state *j*:

$$p_i \triangleq \lim_{t \to \infty} \Pr\{S^t = i | S^1 = j\}.$$
(8)

Therefore, the mean number of new packet arrivals during a slot could be considered, which we refer to as the mean traffic arrival rate:

$$\lambda \triangleq \sum_{i} \lambda_{i} p_{i}. \tag{9}$$

In practice the mean traffic arrival rate could be calculated as the number of newly arrived packets during some sufficiently long interval T related to the duration of this interval, that is:

$$\lim_{T \to \infty} \frac{\sum_{t=0}^{T-1} X^t}{T} = \lambda.$$
(10)

As discussed in Section 1 the DBMAPs are widely used for many practical tasks, including the following. Consider a continuous-time process of packet generation, when packets arrive after intervals $T^{(1)}, T^{(2)}, \ldots, T^{(k)}, \ldots$ (see Fig. 3). Assume that the durations of the intervals $T^{(1)}, T^{(2)}, \ldots, T^{(k)}, \ldots$ are independent and identically distributed (i.i.d.) random variables. The task is to construct an equivalent DBMAP-based description of the considered packet arrival process. We address this task in the rest of the text.

From the DBMAP properties (7)-(9) it follows that:

$$\lambda_1 = \lambda_{ON}; \quad \lambda_2 = 0; \quad \lambda = \frac{\lambda_{ON} \cdot (1 - e^{-\tilde{C}_2})}{2 - e^{-\tilde{C}_1} - e^{-\tilde{C}_2}}; \tag{11}$$
$$p_1 = \frac{1 - e^{-\tilde{C}_2}}{2 - e^{-\tilde{C}_1} - e^{-\tilde{C}_2}}; \quad p_2 = \frac{1 - e^{-\tilde{C}_1}}{2 - e^{-\tilde{C}_1} - e^{-\tilde{C}_2}}.$$



Fig. 3. Equivalent description of the continuous-time arrival process

4 DBMAP-based Model Construction

In this section we construct a DBMAP-based model described in Section 3 corresponding to the realistic IPP-based traffic model recommended by [11], [12] and [13]. We remind that an IPP-based model is characterized by three parameters: C_1 , C_2 and λ_{ON} . The probability that the ON period duration T_{ON} exceeds a threshold value of τ is calculated as

$$\Pr\{T_{ON} > \tau\} = e^{-C_1 \cdot \tau}.$$
(12)

The corresponding probability for the OFF period is

$$\Pr\{T_{OFF} > \tau\} = e^{-C_2 \cdot \tau}.$$
(13)

The mean durations of the ON and the OFF periods are equal to $E[T_{ON}] = 1/C_1$ and $E[T_{OFF}] = 1/C_2$, respectively. We note that the IPP-based model is a continuous-time model, whereas the DBMAP-based model is a discrete-time model. In order to perform the discretization of the IPP the time axis should be slotted. Fig. 4 shows a Markov chain with transition probabilities that correspond to the considered case. We assume that the IPP state cannot change within a slot duration, which holds only for the sufficiently small slot sizes.



Fig. 4. Constructed Markov chain

According to the DBMAP description in Section 3, it is fully described by a set of matrices $\mathbf{B_n}$. In the considered case the size of the matrices is 2×2 . Their elements are obtained as follows:

$$\begin{array}{l} \Pr\{X^t = n, S^{t+1} = ON | S^t = ON\} &= \frac{\lambda_{ON}^n}{n!} \cdot e^{-\lambda_{ON}} \cdot e^{-C_1}.\\ \Pr\{X^t = n, S^{t+1} = ON | S^t = OFF\} &= \frac{\lambda_{ON}^{\overline{n}!}}{n!} \cdot e^{-\lambda_{ON}} \cdot (1 - e^{-C_2}).\\ \Pr\{X^t = n, S^{t+1} = OFF | S^t = OFF\} = 1 \cdot e^{-C_2}, \quad n = 0.\\ \Pr\{X^t = n, S^{t+1} = OFF | S^t = ON\} &= 1 \cdot (1 - e^{-C_1}), \quad n = 0.\\ \Pr\{X^t = n, S^{t+1} = OFF | S^t = OFF\} = 0, \quad n \neq 0.\\ \Pr\{X^t = n, S^{t+1} = OFF | S^t = ON\} &= 0, \quad n \neq 0. \end{array}$$

Now in order to write the set of matrices $\mathbf{B}_{\mathbf{n}}$ explicitly one should substitute the values of n (in the most general case ranging from 0 to ∞) and calculate the respective elements. Below we illustrate the general structure of these matrices:

$$B_0 = \begin{bmatrix} e^{-\lambda_{ON}} \cdot e^{-C_1} & e^{-\lambda_{ON}} \cdot (1 - e^{-C_1}) \\ (1 - e^{-C_2}) & e^{-C_2} \end{bmatrix}.$$
 (14)

$$B_1 = \begin{bmatrix} \lambda_{ON} \cdot e^{-\lambda_{ON}} \cdot e^{-C_1} \lambda_{ON} \cdot e^{-\lambda_{ON}} \cdot (1 - e^{-C_1}) \\ 0 & 0 \end{bmatrix}.$$
 (15)

$$B_{2} = \begin{bmatrix} \frac{\lambda_{ON}^{2}}{2} \cdot e^{-\lambda_{ON}} \cdot e^{-C_{1}} & \frac{\lambda_{ON}^{2}}{2} \cdot e^{-\lambda_{ON}} \cdot (1 - e^{-C_{1}}) \\ 0 & 0 \end{bmatrix}.$$
 (16)

5 Numerical Results

In this section we conduct the performance evaluation of the DBMAP-based traffic model constructed in Section 4. This discrete-time model is equivalent to the the continuous-time IPP model proposed for HTTP traffic simulation in the IEEE 802.16 wireless networks [11]. In order to compare the IPP and the DBMAP input sources we consider the simplest queueing system with the deterministic service time. Therefore, for the discrete case we have a DBMAP/D/1 queue. We assume that the service time is equal to an IEEE 802.16 frame duration (5 ms). In order to use the expressions (14)-(16) and establish the set of matrices $\mathbf{B_n}$ we also make the slot length equal to an IEEE 802.16 frame.

In Fig. 5 we plot the simulated average packed delay for IPP and DBMAP traffic models. The bottom axis ranges possible packet arrival rates. We note that the delay for both models is approximately the same, except for some discrepancy in the near-critical region, which however falls into the confidence interval.

We further investigate the delay distribution for the intermediate normalized packet arrival rate of 0.5. Fig. 6 demonstrates the empirical Probability Density Function (PDF), which verifies that the delay distributions for both traffic models are very close.

Finally, we also show the empirical Cumulative Distribution Function (CDF) in Fig. 7 to confirm that the IPP and the DBMAP input sources demonstrate similar delay performance.



Fig. 5. Packet delay comparison

6 Conclusion

In this work we have conducted a survey of existing traffic models for emerging wireless communication systems. It indicated that there is a disproportion between the models proposed by the standardization bodies and the models that are analyzed in the scientific literature. While the former are often analytically intractable, the latter have limited practical relevance. We fill in this gap by proposing a simple technique to construct a model, which is based on the discrete-time batch Markovian arrival process from the realistic model, which is based on the continuous-time interrupted Poisson process. We also verify that the models demonstrate similar performance in the framework of a simple queueing system. The constructed model shows an example of how the existing theory behind the DBMAP processes could be used to assist the performance evaluation of the real-world networking technologies.

Experimental traffic generation models are generally continuous-time and thus their analysis is somewhat complicated. At the same time, the theory behind the discrete-time processes is much more developed. Therefore, it is important to construct a discrete-time model equivalent to an experimental continuous-time model. In this paper we present such a construction mechanism.

Acknowledgments

This work is supported by the Russian Foundation for Basic Research (projects # 10-08-01071-a and # 08-08-00403-a), as well as by the Branch of Nano- and Information Technologies of Russian Academy of Sciences (project 2.3)



Fig. 6. Empirical PDF of packet delays

References

- D. Moltchanov, "On-line state detection in time-varying traffic patterns," in Proc. of the 7th International Conference on Next Generation Teletraffic and Wire/Wireless Advanced Networking, pp. 49–60, 2007.
- B. Mah, "An empirical model of HTTP network traffic," in Proc. of the 16th Annual Joint Conference of the IEEE Computer and Communications Societies, vol. 2, pp. 592–600, 1997.
- R. Bolla and M. Repetto, "A new model for traffic forecast in wireless networks with fast mobile users," in Proc. of the International Symposium on Performance Evaluation of Computer and Telecommunication System, vol. 35, pp. 153–158, 2003.
- M. Pustisek, I. Humar, and J. Bester, "Empirical analysis and modeling of peerto-peer traffic flows," in *Proc. of the 14th IEEE Mediterranean Electrotechnical Conference*, pp. 169–175, 2008.
- C. Baugh and J. Huang, Traffic model for 802.16 TG3 MAC/PHY simulations. IEEE 802.16 Contribution 802.16.3c-01/30r1, March 2001.
- 6. C. Baugh, 4IPP Traffic Model for IEEE 802.16.3. IEEE 802.16 Contribution 802.16.3c-00/51, October 2000.
- IEEE 802.16-2009. IEEE Standard for Local and metropolitan area networks. Part 16: Air Interface for Broadband Wireless Access Systems, May 2009.
- Y.-G. Jan, Y.-H. Lee, M.-H. Chuang, H.-W. Tseng, J.-Y. Lin, and C.-W. Su, Mean Traffic Bit Rate with ON-SID Modeling of VoIP Traffic. IEEE 802.16 Contribution C802.16m-07/123, July 2007.
- K. D. Turck, S. Andreev, S. D. Vuyst, D. Fiems, S. Wittevrongel, and H. Bruneel, "Performance of the IEEE 802.16e sleep mode mechanism in the presence of bidirectional traffic," in *Proc. of the International Workshop on Green Communica*tions, 2009.
- I. Papapanagiotou, J. Vardakas, G. Paschos, M. Logothetis, and S. Kotsopoulos, "Performance evaluation of IEEE 802.11e based on ON-OFF traffic model," in Proc. of the 3rd International Conference on Mobile Multimedia Communications, 2007.



Fig. 7. Empirical CDF of packet delays

- J. Zhuang, L. Jalloul, R. Novak, and J. Park, *IEEE 802.16m Evaluation Method*ology Document (EMD). IEEE 802.16 Contribution 802.16m-08/004r5, January 2009.
- 12. WiMAX System Evaluation Methodology, December 2007.
- 3GPP2 Contribution C.R1002-0, CDMA2000 Evaluation Methodology, December 2004.
- A. Adas, "Traffic models in broadband networks," *IEEE Communications Maga*zine, vol. 35, no. 7, pp. 82–89, 1997.
- D. Staehle, K. Leibnitz, and P. Tran-Gia, "Source traffic modeling of wireless applications," tech. rep., University of Wurzburg, 2000.
- L. Muscariello, M. Mellia, M. Meo, M. A. Marsan, and R. L. Cigno, "Markov models of Internet traffic and a new hierarchical MMPP model," *Computer Communications*, vol. 28, no. 16, pp. 1835–1851, 2005.
- A. Klemm, C. Lindemann, and M. Lohmann, "Traffic modeling of IP networks using the batch Markovian arrival process," in *Proc. of the 12th International Conference on Computer Performance Evaluation, Modelling Techniques and Tools*, pp. 92–110, 2002.
- D. Lucantoni, "New results on the single server queue with a batch Markovian arrival process," *Stochastic Models*, vol. 7, pp. 1–46, 1991.
- A. Klemm, C. Lindemann, and M. Lohmann, "Traffic modeling and characterization for UMTS networks," in *Proc. of the IEEE Global Telecommunications Conference*, vol. 3, pp. 1741–1746, 2001.
- C. Blondia, "A discrete-time batch Markovian arrival process as B-ISDN traffic model," *Belgian Journal of Operations Research, Statistics and Computer Science*, vol. 32, pp. 3–23, 1993.
- P. Salvador, A. Pacheco, and R. Valadas, "Modeling IP traffic: joint characterization of packet arrivals and packet sizes using BMAPs," *Computer Networks: The International Journal of Computer and Telecommunications Networking*, vol. 44, no. 3, pp. 335–352, 2004.
- J. Zhao, Performance modeling and evaluation of digital video over wireless networks. PhD thesis, Hong Kong University of Science and Technology, 2003.

- J.-A. Zhao, B. Li, and l. Ahmad, "Traffic modeling for layered video," in Proc. of the International Conference on Multimedia and Expo, vol. 1, pp. 497–500, 2003.
- J.-A. Zhao, B. Li, and I. Ahmad, "Bandwidth estimation for wireless video transmission," in *Proc. of the International Conference on Multimedia and Expo*, vol. 3, pp. 1807–1810, 2004.
- 25. S. Baey, "Modeling MPEG4 video traffic based on a customization of the DBMAP," in Proc. of the International Symposium on Performance Evaluation of Computer and Telecommunication Systems, pp. 705–714, 2004.
- D. Fiems, V. Inghelbrecht, B. Steyaert, and H. Bruneel, "Markovian characterisation of H.264/SVC scalable video," in Proc. of the 15th International Conference on Analytical and Stochastic Modeling Techniques and Applications, pp. 1–15, 2008.
- 27. K. D. Turck, S. D. Vuyst, D. Fiems, and S. Wittevrongel, "An analytic model of IEEE 802.16e sleep mode operation with correlated traffic," in *Proc. of the* 7th International Conference on Next Generation Teletraffic and Wire/Wireless Advanced Networking, pp. 109–120, 2007.
- K. D. Turck, S. D. Vuyst, D. Fiems, and S. Wittevrongel, "Performance analysis of the IEEE 802.16e sleep mode for correlated downlink traffic," *Telecommunication Systems*, vol. 39, no. 2, pp. 145–156, 2008.
- B. V. Houdt and C. Blondia, "Stability and performance of stack algorithms for random access communication modeled as a tree structured QBD Markov chain," *Stochastic Models*, vol. 17, no. 3, pp. 247–270, 2001.
- B. V. Houdt and C. Blondia, "Robustness of Q-ary collision resolution algorithms in random access systems," *Performance Evaluation*, vol. 57, no. 3, pp. 357–377, 2004.
- B. V. Houdt and C. Blondia, "Throughput of Q-ary splitting algorithms for contention resolution in communication networks," *Journal of Communications in Information and Systems*, vol. 4, no. 2, pp. 135–164, 2005.
- 32. A. Alfa and S. Chakravarthy, "A discrete queue with the markovian arrival process and phase type primary and secondary services," *Stochastic Models*, vol. 10, no. 2, pp. 437–451, 1994.
- 33. A. Alfa, K. Dolhun, and S. Chakravarthy, "A discrete single server queue with Markovian arrivals and phase type group services," *Journal of Applied Mathematics* and Stochastic Analysis, vol. 8, no. 2, pp. 151–176, 1995.
- C. Herrmann, "The complete analysis of the discrete time finite DBMAP/G/1/N queue," *Performance Evaluation*, vol. 43, pp. 95–121, 2001.
- J.-A. Zhao, B. Li, X.-R. Cao, and I. Ahmad, "A matrix-analytic solution for the DBMAP/PH/1 priority queue," *Queueing Systems: Theory and Applications*, vol. 53, no. 3, pp. 127–145, 2006.