Binary Exponential Backoff Algorithm Analysis in the Lossy System with Frames

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Abstract

In this paper we consider a random multiple-access system under the control of the binary exponential backoff (BEB) algorithm. We account for the time frame structure, as well as for the possibility of a data loss. An alternative approach to calculating the BEB algorithm saturation rate is demonstrated and extended for the above system features. The results of this work may be used for optimizing BEB parameters at the bandwidth reservation stage in IEEE 802.16 and at the data transmission stage in IEEE 802.11 telecommunication standards.

I. Introduction and Background

Following the traditional approach to the design and development of a telecommunication system, each pair of communicating users was supplied with a dedicated communications channel. However, when the use of such a design is economically impractical, impossible or dynamic topology support is required, so-called multiple-access channels are organized [1], [2], which are collectively utilized by a set of channel users. In recent decades the number of such systems increases dramatically, which substantiates the need for deeper analytical study of the multiple-access environments.

In particular, the above is relevant for the local and metropolitan area networks, where the use of the wireless technology enables the flexibility of the network topology, including the support for mobile users, high development speed and low deployment cost. The unrivaled leaders among the technologies utilizing multiple-access communication channels are metropolitan area networks standardized by IEEE 802.16 [3] and local area networks standardized by IEEE 802.11 [4]. The former standard defines a centralized data network, whereas the latter specifies a distributed network, in which there could be no central coordinating node.

The use of a multiple-access channel imposes several restrictions on the design of the upper system layers. In particular, a special multiple-access algorithm is necessary to control the access of the system users to the communications channel. A wide variety of such algorithms is known to operate in both centralized and distributed telecommunication networks. In a centralized network a transmission schedule is typically created, which is preceded by the bandwidth reservation stage. Accounting for the relatively low arrival rate of the bandwidth requests, as well as their bursty nature random multiple-access (RMA) algorithms are used to send them. In these conditions an RMA algorithm is known to guarantee the lower mean request delay. The combination of an RMA algorithm at the bandwidth reservation stage and a scheduling algorithm at the data packet transmission stage is currently used in many profiles of IEEE 802.16 standard. In IEEE 802.11 standard an RMA algorithm directly arbitrates the transmission of the data packets.

Performance evaluation of the randomized profiles of IEEE 802.16 standard has become a subject of many research papers. In particular, the bandwidth reservation stage is often considered, when users send bandwidth requests to obtain a portion of channel resources.

A detailed description of the various reservation techniques was given as long ago as in [5]. IEEE 802.16 adopts the notorious binary exponential backoff (BEB) algorithm as the means of collision resolution between the requests. Notably, IEEE 802.11 specifies the same algorithm to resolve collisions between the data packets themselves. Therefore, the performance analysis of the BEB algorithm is an attractive research area.

The asymptotic behavior of BEB algorithm was thoroughly investigated in the scientific literature. In [6] it was shown that the BEB algorithm is unstable in the framework of the RMA system with infinite population of users, that is it has zero transmission rate. By contrast, in [7] the BEB algorithm is proved to be stable for the sufficiently low arrival rates and finite user population. It is known that the infinite population model allows for demonstrating the ultimate performance limitation of an algorithm, whereas the finite population model gives bounds on its practical applicability.

Finally, for the finite population model the BEB algorithm was put in so-called saturation conditions by [8] and [9], where useful Markov chains were constructed to evaluate its performance. In saturation conditions every system user always has pending data to transmit. Therefore, saturation is the highest possible system load scenario for the given number of users M. Moreover, it allows for establishing the maximum performance characteristics of the algorithm and for avoiding the consideration of a certain arrival flow type. In this paper we extend the saturation model of [8] and [9] for a more practical scenario. The contribution of this work is twofold. Firstly, it presents an alternative regenerative process approach to analyze the saturation performance of the BEB algorithm, which is believed to be simpler than the techniques used by authors of [8] and [9]. Secondly, the system model is extended to account for IEEE 802.16 system features, such as frame structure and the possibility of a packet loss.

II. System Model

Below we develop a system model based on the well-known RMA model from [2], [10], [1], [11]. We are primarily interested in the performance analysis of the bandwidth reservation mechanism in IEEE 802.16. However, the model is also applicable for IEEE 802.11 system. The respective modifications are also discussed below.

Assumption 1. *The system.* There are M users in the system and the system time is broken into equal time intervals. The length of each interval is exactly the duration of an IEEE 802.16 frame. Each frame incorporates K equal contention slots (transmission opportunities) for sending bandwidth requests. Users send bandwidth requests to contend for the channel resources. We leave the packet transmission stage out of scope of this paper.

Assumption 2. *The channel.* In each contention slot one and only one of the following events may occur:

- only one user sends a request (”SUCCESS”);
- no user sends a request (”EMPTY”);
- two or more users send requests in the same slot, which leads to the corruption of the collided requests. (”COLLISION”).

The communication channel is noise-free.

Assumption 3. *The feedback.* The narrowest feedback of type ”SUCCESS”–”NON-SUCCESS” is available to a user in those slots, where it has transmitted a request. That is, a user only ”knows” whether its own transmission was successful or not. This feedback size is sufficient for the operation of the BEB algorithm. The algorithms residing on such a feedback type are often referred to as the acknowledgment-based. The feedback is faultless and is available to a user by the end of a frame (by the beginning of the next frame), that is once in K contention slots.

Assumption 4. *The user.* Each system user has a buffer to store a single bandwidth request. Following by [8] and [9] we assume that this buffer is always non-empty, that is a user always has a pending bandwidth request. Thus, the saturation conditions are introduced.

The above model may be modified to reflect the features of IEEE 802.11 standard. In particular, there are no frames defined by the standard and $K = 1$. The users send data packets instead of bandwidth requests. More importantly, the channel events ("SUCCESS", "EMPTY", "COLLISION") have variable duration instead of one contention slot length in IEEE 802.16. Therefore, the model should be extended for the unequal slot sizes. This extension, however, does not cause significant difficulty and has been first presented in [8]. Therefore, the described system model may be tailored for the contention access in IEEE 802.11 standard.

We define an RMA algorithm in the considered system as a rule, according to which a user with a pending bandwidth request decides whether to transmit this request in the next contention slot, defer its transmission or discard the request. If a request is discarded the corresponding data packet is lost [12]. As such, we introduce the lossy system, where data packet loss is possible. If, for some reason, a user decides not to discard requests, the system becomes lossless. The lossy system, however, is expected to be more practical than the lossless one, since in the real-world wireless networks, if the channel is jammed with collisions the data packets are often discarded by users to clear it. We additionally assume that such a discard is made after Q unsuccessful retransmission attempts. Q is, thus, another parameter of the system model, which represents the maximum number of bandwidth request retransmissions. Clearly, for the lossless case $Q \to \infty$.

III. Proposed Analysis

A. Truncated BEB algorithm

Here we describe the rules of the BEB algorithm specified by IEEE 802.16 standard. A user defers the first bandwidth request transmission attempt for the random number of contention slots sampled uniformly in the range $\{0, 1, \ldots, W_0 - 1\}$, where $W_0 \geq 1$ is the initial contention window. The sampled number is often referred to as the backoff counter S, since it is decreased after each slot, for which a user defers the transmission. As a consequence of an unsuccessful transmission attempt a user doubles the contention window value and chooses a new contention slot for the request retransmission in the range $\{0, 1, \ldots, 2^B W_0 - 1\}$, where B is the user backoff stage (number of the current retransmission attempt, number of collisions suffered by the current request).

In practice, to avoid the unlimited growth of the contention window $W_i = 2^i W_0$, it is bounded from above by the value $W_m = 2^m W_0$, where m is the maximum backoff stage. Thus, upon an unsuccessful transmission attempt a user doubles its contention window value unless it does not exceed W_m. Therefore, the sampling range changes to $\{0, 1, \ldots, 2^{\min(B,m)} W_0 - 1\}$. This truncated modification of the BEB algorithm is adopted by IEEE 802.16 standard (see Fig. 1). Notice, that IEEE 802.16 standard [3] does not specify the relationship between the parameters W_0, m and K. For instance, if $W_0 < K$ then at the first transmission attempt some contention slots will never be used. For this reason we set $W_0 = lK$, where l is a natural number ($l \geq 1$). As such, the (re)transmission attempts are evenly distributed across the available contention slots. In Fig. 1 $l = 1$.

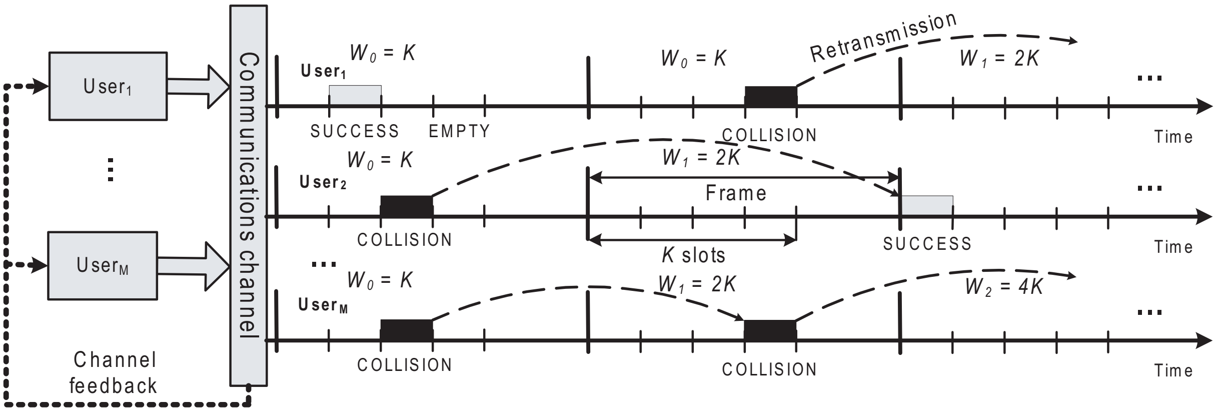

Fig. 1. BEB algorithm operation in IEEE 802.16 standard

B. Lossless system

As discussed above, to describe the lossless system we let the maximum number of retransmission attempts Q go to infinity. Thus, the bandwidth requests are never discarded and, consequently, data packets are never lost.

Generally, RMA algorithms are characterized by two performance metrics: their transmission rate and the mean packet delay. Remember, that we intend to analyze the performance of the BEB algorithm in the saturation conditions. Therefore, the notion of the delay is irrelevant, since in this case it is theoretically infinite. Moreover, the transmission (or, output) rate $R_{BEB}(K)$ is traditionally defined [10] as the supremum of the arrival rates that guarantee finite mean packet delay. In saturation conditions there is no arrival flow and thus the main performance metric is the maximum stationary success probability. Generally, it is unclear whether this probability has the same value as the transmission rate of the unsaturated system with some arrival rate. In [8] and [9] this fact was accepted intuitively, but its strict proof followed only in [13]. For simplicity of the notation we further refer to the maximum stationary success probability as to the BEB algorithm rate, despite the fact that there is no arrival flow.

The BEB algorithm rate in the saturation conditions for the lossless system with frames was obtained in [14], where a corresponding Markov chain was constructed. The same result, however, may be obtained with a more simple technique of regenerative processes. In [8], [9] and [14] the same assumptions were adopted that allow for substituting the analysis of the entire RMA system with the analysis of a single tagged user. Thus, we also assume that at each (re)transmission attempt regardless of the number of the previous attempts each bandwidth request collides with constant probability p_c, which is referred to as the conditional collision probability, or the probability that is ”seen” by the transmitted request. Furthermore, we consider the probability of a request (re)transmission in a randomly chosen contention slot p_t and similarly assume that it is constant. The (re)transmission attempts of a user in the saturation conditions are assumed to be independent.

Accounting for the fact that the bandwidth request of the tagged user collides if and only if at least one of the remaining $M-1$ users (re)transmits its request in the same contention slot, we obtain:

$$p_c = 1 - (1 - p_t)^{M-1}. \tag{1}$$

In order to establish p_t as a function of p_c, we formulate and prove a set of auxiliary propositions.

Proposition 1. *We introduce a random variable $Z^{(i)}$, which is equal to 1 if the tagged user transmits its bandwidth request in a contention slot number i and is equal to 0 otherwise. The time instants in which the "SUCCESS" feedback from the transmission is received by the tagged user are the regeneration points of the stochastic processes given by the sequence $Z^{(i)}$.*

This proposition holds due to the above assumptions. Firstly, we consider the RMA system in the saturation conditions where there is no arrival flow to the tagged user. Secondly, in the time instants corresponding to the reception of the "SUCCESS" feedback the collision counter of the tagged user is reset to zero and the contention window W_0 is reset to its initial value. Therefore, the tagged user itself re-enters the initial state at the considered time instants.

Proposition 2. *The bandwidth request (re)transmission probability p_t by the tagged user may be obtained as:*

$$p_t = \lim_{n\to\infty} \frac{\sum\limits_{i=1}^{n} B^{(i)}}{\sum\limits_{i=1}^{n} D^{(i)}} = \frac{E[B]}{E[D]}, \tag{2}$$

where $B^{(i)}$ is the number of request (re)transmissions in the regeneration cycle number i and $D^{(i)}$ is the duration of the regeneration cycle number i, being expressed in the contention slots.

This proposition may be proved as follows. According to the proposition 1 the regeneration points of the stochastic process $Z^{(i)}$ correspond to the beginnings of the respective contention periods within subsequent frames. Remember, that each contention period incorporates K contention slots. The value of p_t may be calculated as $p_t = \lim\limits_{n\to\infty} \frac{\sum_{i=1}^{n} B^{(i)}}{\sum_{i=1}^{n} D^{(i)}}$. Therefore, (2) immediately follows from the regenerative character [15] of the underlying process. The values of $B^{(i)}$ and $D^{(i)}$ are independent and identically distributed, therefore, we can omit the upper indexes i in (2).

Proposition 3. *The bandwidth request (re)transmission probability p_t by the tagged user of the lossless system may be obtained as:*

$$p_t = \frac{2(1-2p_c)}{(1-2p_c)(W_0+K) + p_c W_0 (1-(2p_c)^m)}. \tag{3}$$

Proof: The value of B conforms to the geometric distribution with the parameter $1-p_c$, which implies:

$$E[B] = \sum_{i=1}^{\infty} i \Pr\{B=i\} = (1-p_c) \sum_{i=1}^{\infty} i p_c^{i-1} = \frac{1}{1-p_c}. \tag{4}$$

Mathematical expectation of the regeneration cycle duration D may be analogously written as:

$$E[D] = \sum_{i=1}^{\infty} D(i) \Pr\{D=i\} = (1-p_c) \sum_{i=1}^{\infty} D(i) p_c^{i-1}, \tag{5}$$

where $D(i)$ is the regeneration cycle duration, conditioning on the fact that exactly i transmission attempts were made.

Above we set the initial contention window size to $W_0 = lK$, where l is the natural number ($l \geq 1$). Thus, $l = \frac{W_0}{K}$. Consider the value of $D'(i)$ for the case when $1 \leq i \leq m+1$. It could be found as:

$$D'(i) = K \sum_{j=0}^{i-1} \frac{2^j l + 1}{2}. \tag{6}$$

Reorganizing (6), we get:

$$D'(i) = \frac{lK}{2} \sum_{j=0}^{i-1} 2^j + \frac{K}{2} \sum_{j=0}^{i-1} 1 = 2^{i-1} W_0 - \frac{W_0 - iK}{2}. \tag{7}$$

Now we consider the value of $D''(i)$ for the case when $i > m+1$:

$$D''(i) = K \left(\sum_{j=0}^{m} \frac{2^j l + 1}{2} + \sum_{j=m+1}^{i-1} \frac{2^m l + 1}{2} \right). \tag{8}$$

In turn, reorganizing (8), we obtain:

$$\begin{aligned} D''(i) = 2^m W_0 - \frac{W_0 - (m+1)K}{2} + (i - m - 1)K \frac{2^m l + 1}{2} = \\ = 2^{m-1} W_0 (i - m + 1) - \frac{W_0 - iK}{2}. \end{aligned} \tag{9}$$

Thus, for the value of $D(i)$ it holds the following:

$$D(i) = \begin{cases} 2^{i-1} W_0 - \frac{W_0 - iK}{2}, \text{ if } 1 \leq i \leq m+1, \\ 2^{m-1} W_0 (i - m + 1) - \frac{W_0 - iK}{2}, \text{ if } i > m+1. \end{cases} \tag{10}$$

We may now calculate the expected value of $E[D]$ by (5) using (10):

$$E[D] = \frac{(1 - 2p_c)(W_0 + K) + p_c W_0 (1 - (2p_c)^m)}{2(1 - 2p_c)(1 - p_c)}. \tag{11}$$

In order to obtain the sought value of p_t we account for (2). Here $E[B]$ follows from (4) and $E[D]$ is given by (11), which concludes the proof. ∎

It is interesting to mention that (3) accords with the result in [14] and for the special case of no frames in the system ($K = 1$) yields another known result [8]. Equations (1) and (3) constitute a non-linear system with two unknowns, p_c and p_t, which could be solved numerically. Finally, the transmission rate $R_{BEB}(K)$ is calculated as the probability of a single transmission in a contention slot:

$$R_{BEB}(K) = M p_t (1 - p_t)^{M-1}. \tag{12}$$

C. Lossy system

The above analytical approach to calculating the BEB algorithm transmission rate for the lossless system may be extended for the lossy case. Here we again consider the maximum number of bandwidth request retransmissions Q. Therefore, the maximum number of request transmissions is $Q+1$ and according to the proposition 2 (2) is still valid for the bandwidth request (re)transmission probability p_t by the tagged user.

Proposition 4. *The bandwidth request (re)transmission probability p_t by the tagged user of the lossy system may be obtained as:*

$$p_t = \frac{2(1-2p_c)(1-p_c^{Q+1})}{W_0(1-p_c)(1-(2p_c)^{Q+1}) + K(1-2p_c)(1-p_c^{Q+1})}, \quad \text{if } Q \le m \text{ and} \tag{13}$$

$$p_t = \frac{2(1-2p_c)(1-p_c^{Q+1})}{(1-2p_c)(W_0(1-2^m p_c^{Q+1}) + K(1-p_c^{Q+1})) + p_c W_0(1-(2p_c)^m)}, \quad \text{if } Q > m.$$

Proof: Here we use the same notation as in the propositions 2 and 3. Firstly, we find the average number of the request (re)transmissions $E[B]$ in a regeneration cycle as:

$$E[B] = \sum_{i=1}^{Q+1} i \Pr\{B=i\} = \frac{1-p_c^{Q+1}}{1-p_c}. \tag{14}$$

Further we calculate the expected regeneration cycle duration $E[D]$ analogously to (5):

$$E[D] = (1-p_c)\sum_{i=1}^{Q+1} D(i) p_c^{i-1} + p_c^{Q+1} D(Q+1), \tag{15}$$

where $D(i)$ is again the conditional regeneration cycle duration.

It is easy to show that for the cases when $1 \le i \le m+1$ and $i > m+1$ the expressions for $D(i)$ in the considered system coincide with those in the lossless case (7) and (9). It follows from the fact that the rules of the BEB algorithm has remained unchanged. In order to calculate $E[D]$ using (15) we have to consider two cases: $Q \le m$ and $Q > m$. We begin with the first one as:

$$E[D'] = (1-p_c)\left[\sum_{i=1}^{Q+1}\left(2^{i-1}W_0 - \frac{W_0 - iK}{2}\right)p_c^{i-1}\right] + \\ + p_c^{Q+1}\left(2^Q W_0 - \frac{W_0-(Q+1)K}{2}\right). \tag{16}$$

The probability of a request (re)transmission p_t for the considered case is given by substituting $E[B]$ from (14) and $E[D]$ from (16) into (2). After elementary transformations, we establish:

$$p'_t = \frac{2(1-2p_c)(1-p_c^{Q+1})}{W_0(1-p_c)(1-(2p_c)^{Q+1}) + K(1-2p_c)(1-p_c^{Q+1})}. \tag{17}$$

Now we address the case of $Q > m$ and calculate the respective value of $E[D'']$:

$$E[D''] = (1 - p_c)\left[\sum_{i=1}^{m+1}\left(2^{i-1}W_0 - \frac{W_0 - iK}{2}\right)p_c^{i-1} + \right.$$
$$\left. + \sum_{i=m+2}^{Q+1}\left(2^{m-1}W_0(i - m + 1) - \frac{W_0 - iK}{2}\right)p_c^{i-1}\right] +$$
$$+p_c^{Q+1}\left(2^{m-1}W_0(Q - m + 2) - \frac{W_0 - (Q+1)K}{2}\right). \tag{18}$$

The sought probability p_t for this case is given by substituting $E[B]$ from (14) and $E[D]$ from (18) into (2). Again, after elementary transformations, we get:

$$p_t'' = \frac{2(1 - 2p_c)(1 - p_c^{Q+1})}{(1 - 2p_c)(W_0(1 - 2^m p_c^{Q+1}) + K(1 - p_c^{Q+1})) + p_c W_0(1 - (2p_c)^m)}. \tag{19}$$

Thus, (13) immediately follows from (17) and (19). ■

Notice, that (13) is a generalization of (3) for the lossy system case. Therefore, (1) and (13) still constitute a non-linear system with two unknowns p_c and p_t. It may be solved numerically to establish the BEB transmission rate analogously to (12) as:

$$R_{BEB(Q+1)}(K) = Mp_t(1 - p_t)^{M-1}. \tag{20}$$

IV. Numerical Results and Conclusion

A. Lossless system

The proposed approach allows for the optimization of the bandwidth request (re)transmission probability, which leads to the maximization of the BEB transmission rate over all the pairs of its parameters (W_0, m). It may be shown that the rate is maximized when $m = 0$. Below we consider the optimal system in more detail.

Substituting $m = 0$ into (3) it is easy to see that $p_t = \frac{2}{W_{opt}+K}$, where W_{opt} is the optimal contention window size. Notice, that (12) reaches its maximum value for $Mp_t = \frac{2M}{W_{opt}+K} = 1$. Therefore, W_{opt} may be calculated as:

$$W_{opt} = 2M - K. \tag{21}$$

However, the use of the optimal contention window size in IEEE 802.16 standard is difficult as it could be not an integer power of 2 [3]. As an example, in Fig. 2 the BEB transmission rate is shown as a function of the maximum backoff stage m for the different values of W_0. For the considered scenario with $M = 40$ and $K = 8$ W_{opt} is 72. At the same time the BEB algorithm with the parameters $W_0 = 32$ and $m = 2$ has slightly lower rate than the optimal algorithm. Consequently, this suboptimal BEB algorithm may be used in IEEE 802.16 for $M = 40$ and $K = 8$.

B. Lossy system

The dependence of the BEB algorithm transmission rate in the lossy system on the maximum number of bandwidth request transmissions ($Q + 1$) for $M = 40$ and $K = 8$ is shown in Fig. 3. For the optimal algorithm with $W_{opt} = 72$ and $m = 0$ the rate is independent of Q, which is in contrast to the suboptimal algorithm with $W_0 = 32$ and $m = 2$.

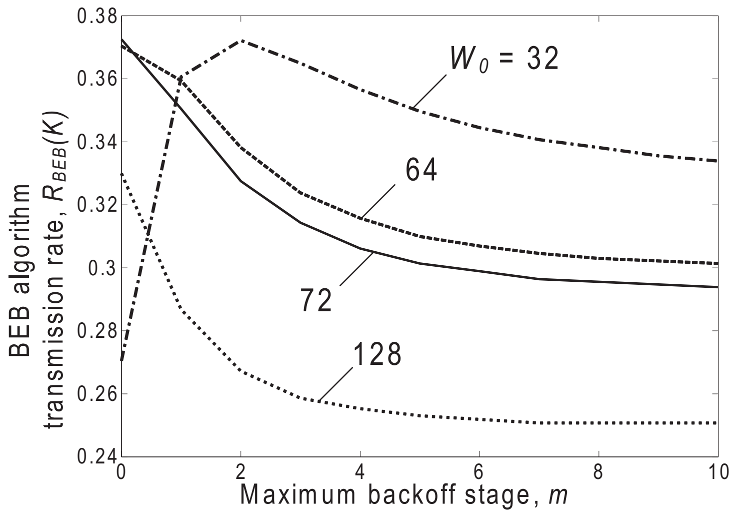

Fig. 2. BEB algorithm transmission rate in the lossless system for $M = 40$ and $K = 8$

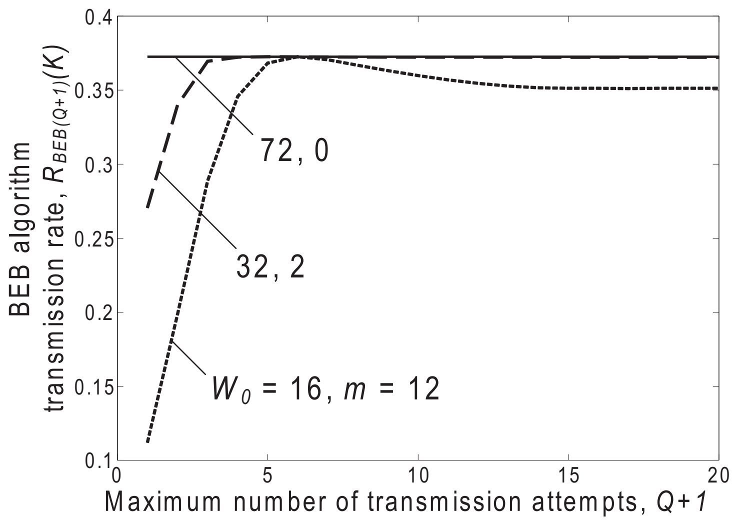

Fig. 3. BEB algorithm transmission rate in the lossy system for $M = 40$ and $K = 8$

In Fig. 3 one may notice that for $Q = 4$ there is a maximum of the transmission rate for the suboptimal algorithm, which coincides with the rate of the optimal algorithm. This observation leads to the following statement. There exists a value of Q, for which the BEB algorithm transmission rate in the lossy system is equal to that in the lossless system. This statement may be proved using propositions 3 and 4. It is easier to demonstrate for $W_0 = 16$ and $m = 12$ as the maximum of the transmission rate function is more clear and is reached for $Q = 5$.

C. Conclusion

In this paper we introduced a more practical lossy system and extended the notorious approach to the saturation analysis of the binary exponential backoff random multiple-access algorithm for this system. Also we proposed an alternative technique to calculating the transmission rate of the considered algorithm by using its regenerative property. Our analysis allows for the optimization of the algorithm's parameters in IEEE 802.16 and IEEE 802.11 telecommunication standards.

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