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TREE ALGORITHMS WITH FREE ACCESS AND INTERFERENCE CANCELLATION IN PRESENCE OF CANCELLATION ERRORS

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ABSTRACT

In this paper we address a promising cross-layer MAC-PHY technique that combines both tree algorithm (TA) for collision resolution and successive interference cancellation (SIC) scheme to recover the signals of collided users. The resulting SICTA protocol family is known to achieve higher throughputs in comparison to the conventional MAC protocols. We concentrate on the performance evaluation of SICTA protocols with a single memory location. Furthermore, we extend the system model to account for possible interference cancellation errors at the receiver. The existing SICTA protocol with free access is then presented and enhanced to acquire resistance to cancellation errors. The throughput and delay of both initial and resulting protocols are compared via simulation to demonstrate the feasibility of the proposed solution.

I. INTRODUCTION

Media access control (MAC) protocols are dedicated to arbitration of shared broadcast channel access by a population of channel users. The random MAC protocols provide attractive performance characteristics, such as low packet delay and relatively high throughput, especially when the network traffic is bursty [1], [2]. Typically, each random MAC protocol incorporates both *channel access algorithm* (CAA) and *collision resolution algorithm* (CRA).

The CAA specifies the broadcast channel access procedure, whereas the CRA defines rules to resolve packet collisions (i.e. simultaneous transmissions of two or more data packets), whenever they arise. In ALOHA and ALOHA-based protocols, such as *diversity slotted aloha* (DSA), *binary exponential backoff* (BEB), *carrier sense multiple access* (CSMA) and others no particular CRA is specified. These protocols are generally easy to implement and once a collision occurs their underlying idea is to defer the subsequent packet retransmission to some future time in a 'hope' that the communications channel becomes idle.

Tree algorithm (TA) simultaneously proposed in [3] and [4] provides an attractive alternative to the Aloha-based schemes as it directly controls the collision resolution process, which guarantees higher *maximum stable throughput* (MST). The first known *standard tree algorithm* (STA) was slightly improved by the *modified tree algorithm* (MTA). The STA and the MTA protocols are jointly referred to as *conventional* TAs below. These protocols received much attention and were addressed by telecommunications standards, such as DAVIC/DVB [5], IEEE 802.14 [6] and DOCSIS [7].

The conventional cross-layer OSI interaction assumes that once a collision of two or more data packets occurs in the communications channel no meaningful data could be recovered from it. However, for wireless communications channel a *successive interference cancellation* (SIC) technique is known to recover the signals of collided users at the *physical* (PHY) layer. More specifically, SIC first tries to detect and demodulate the strongest user signal currently present in the composite captured signal. After it is done, this signal contribution to the original signal is recreated and subtracted from it. A new composite signal is thus produced, which could be again the input for the iterative SIC procedure.

In [8] it was shown that SIC is capable of approaching the theoretical limits for an AWGN channel and its *cancellation error* was introduced and estimated. A cancellation error for a user is defined as its residual signal in the remaining composite signal after the subtraction of the recreated signal. The main reasons of the cancellation error are imperfect (amplitude and phase) channel estimation and incorrect bit decisions. Despite this disadvantage SIC technique has low implementation complexity [9], which impacts to its usage in the interference cancellation receivers [10].

In [11] a novel concept of *successive interference cancellation in a tree algorithm* (SICTA) was first proposed to combine the advantages of both SIC and conventional TAs. A new protocol was described and analyzed that adopts SIC to reuse the collision signals that are stored in (potentially) unbounded memory. The MST of the proposed protocol was shown to be 0.693 in the *reference information theoretical model* ([1], [2]) and proved optimal over the set of all d-ary collision resolution trees in [12]. Notice that the indicated value is two times higher than the MST of the STA protocol (0.346) and sufficiently exceeds the MST of the MTA protocol (0.375).

We emphasize that the aforementioned SICTA modifications theoretically required unbounded memory storage for the received collision signals. By contrast, in [13] the performance of a novel TA with SIC property was investigated for which a single memory location suffices. This protocol is, clearly, more feasible from the implementation point of view. The MST of the proposed protocol was shown to be 0.5698 when using the additional control field/bit with separate feedback, indicating whether the packet is transmitted for the first time. Therefore, this MST value should not be compared with the MST of SICTA as the system model had been changed.

In this paper we propose our own SICTA protocol that requires only a single memory location like in [13], but without the additional feedback. The proposed protocol thus fully conforms to the reference information theoretical model we describe and extend in Section II. Section III describes the core idea of the existing conventional and SICTA protocols. The proposed protocol is presented in Section IV, while its MST and delay are evaluated in Section V.

II. SYSTEM MODEL

The reference information theoretical model described by [1] and [2] have long become a de-facto framework to analyze the performance characteristics of the random multiple access protocols. Following [1], [2] and [14] we present a set of assumptions about the way the packets arrive into the system and are transmitted.

Assumption 1. The system time is slotted into equal slots. The duration of each slot is a unit of the system time, which is exactly the transmission time of one data packet. Each slot is assigned an integer nonnegative number and number t slot corresponds to the time interval of $[t, t+1)$. Hereinafter we refer to number t slot simply as slot t for the sake of brevity. Slot borders are known to all the users and each user is restricted to start its packet transmission only in the beginning of a slot.

Assumption 2. In each slot any and only one of the following error-free events may occur (channel-PHY feedback):
- only one user transmits (**S** event - success);
- none of the users transmit (**E** event - empty);
- two or more users transmit (**C** event - collision).

Assumption 3. When monitoring the channel activity the MAC layer of a user is notified of the channel event by the end of the current slot. The size of the PHY-MAC feedback (see Fig. 1) a user receives is subject to some variation depending on the protocol and is discussed in the following section.

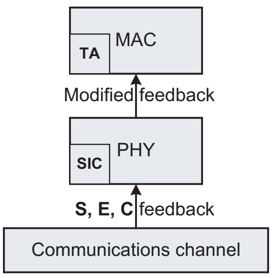

Figure 1: Various feedback types.

Assumption 4. There is an infinite user population, generating packets that are assumed to be unique. Each user is supplied with a buffer sufficient to store only one packet. The packet is stored from the instant of time it arrived into the system to the instant of time it is successfully transmitted. Packet inter-arrival times are assumed to be statistically independent random variables which are distributed exponentially with the mean value of $\frac{1}{\lambda}$. Thus, λ is the arrival rate of the new packets into the system. Notice, that infinite population provides a pessimistic estimation for a finite population system by considering each packet to be a virtual station.

Below we extend the reference model to account for the practical issues of the SIC scheme implementation.

Assumption 5. The receiver is able to store a single signal for which a single memory location is dedicated.

In practical SIC schemes a cancellation error is possible [8], which is the residual signal in the remaining signal after the SIC procedure. For instance, after the cancellation of the signal A from the composite signal $X_A + X_B$ the resulting signal contains $\tilde{Y} = X_B + N_A$, where N_A is the residual signal A. After the subsequent cancellation of signal B we similarly obtain $\tilde{\tilde{Y}} = N_A + N_B$. If the $N_A + N_B$ energy level is sufficiently high the receiver incorrectly decides that the slot is not empty, but rather there is a collision between the nonexistent users. We, therefore, assume that due to cancellation errors the interference cancellation is *unsuccessful* with some constant probability. That is, with this probability receiver obtains a meaningless signal after the next interference cancellation. In practice this probability could be derived as the worst-case estimate of the SIC operation.

Assumption 6. The interference cancellation is imperfect and we distinguish between two possible options. First is that after the *successful* signal is canceled from the composite signal the resulting signal contains a meaningless signal with probability q (see discussion above and example in Fig. 2(c), slot 2). Second is that after the *collision* signal is canceled from the composite signal the resulting signal contains a meaningless signal with probability q' (see example in Fig. 2(e), slot 2). We expect that in practice $q' \geq q$, that is, it is more difficult to cancel the collision signal than the successful signal.

III. EXISTING PROTOCOLS

Each conventional TA may incorporate one of three alternative CAAs, which are *gated* access, *window* access or *free* access. According to the gated CAA the new data packets that arrive during the so-called *collision resolution interval* (CRI) are deferred. Once current collision is resolved, all the deferred packets are transmitted simultaneously to create the following collision and to start a new CRI. Window CAA is a generalization of the gated access scheme for the case when a new CRI is formed not by all the packets that arrive during the previous CRI, but by those arriving within a specified time window.

The gated and the window CAAs are jointly referred to as *blocked access* schemes. By contrast, free (or, *non-blocked access*) CAA assumes the packet is transmitted immediately following its arrival. Therefore, no CRI is formed and this scheme is the easiest to implement since a new channel user may join the existing system operation without the need to monitor the (potentially) entire channel history. Although in this paper we concentrate on the performance evaluation of the free access SICTA modifications, we begin with the brief description of the simplest gated access protocols in order to clarify the SICTA concept.

The conventional STA with gated access requires only 'collision' - 'no collision' PHY-MAC feedback from the receiver (see **Assumption 3**) for its proper operation. New data packets that arrive during the previous CRI are transmitted in the first slot of the following CRI. If 'no collision' feedback is received for this initial slot, the CRI ends. Otherwise, each of the collided users flips a (biased) coin to choose the right subset with probability p and the left subset with probability $1-p$ (see Fig. 2(a)). The procedure is repeated recursively until all the packets are received successfully. The example collision resolution tree in Fig. 2(a) comprises 7 slots, which is exactly the CRI length. For binary STA fair splitting with $p = 0.5$ is optimal and yields the MST of 0.346 ([2], [15]), where MST is formally defined as the highest possible (Poisson) arrival rate that still yields a finite packet delay with probability one.

We notice that in the example collision resolution tree (see Fig. 2(a)) the collision in slot 5 is deterministic, since the collision in slot 3 is followed by the empty right slot 4, which means that all the collided users have chosen the left subset. In [3] and [4] a 'level skipping' was proposed to omit slot 5 and proceed directly to the next tree level. Therefore, the CRI length reduces to 6 slots (see Fig. 2(b)) instead of 7. The conventional binary MTA with gated access uses this idea and results in the MST of 0.375 for fair splitting. However, a biased splitting with $p = 0.582$ is optimal and yields the MST of 0.381 [15]. We finally notice that MTA requires the extended ternary 'success' - 'empty' - 'collision' PHY-MAC feedback to enable the discussed improvement.

In the original SICTA protocol the receiver is supplied with unbounded memory to store the collision signals. Consider the example in Fig. 2(c) where the CRI length is only 4 slots. The contents of the corresponding left slot is determined by canceling the interference from slot 1 after the successful reception of signal C in slot 2. Following the notation from [11], we denote the cancellation procedure as $\tilde{Y}_1 = Y_1 - X_C$. As slot 3 is empty the corresponding left slot is skipped following the rules of the MTA. Finally, the successful reception of signal B in slot 4 immediately yields the recovery of signal A by $\tilde{\tilde{Y}}_1 = \tilde{Y}_1 - X_B = Y_1 - X_C - X_B$.

As the entire left subtree of the STA is omitted by the original SICTA protocol, it results in the MST of 0.693, which is twice the MST of the STA. Additionally, SICTA requires extended k - 'empty' - 'collision' PHY-MAC feedback, where k is the number of successfully decoded packets plus the number of left slots identified as being empty after the SIC procedure.

Free access allows for the easier implementation of a MAC protocol since new users may join the system without the prior monitoring of the channel history. In Fig. 2(d) the performance of the MTA protocol with free access is illustrated. Its main difference from the gated access MTA protocol is that the newly arrived data packets are transmitted immediately and thus influence the collision resolution process. In the example signal D transmitted in slot 5 collides with signal B and, therefore, lengthens the collision resolution. However, the use of free access may also be beneficial if a newly arrived packet is transmitted in the empty slot, e.g. slot 4.

Supplying the receiver with the unbounded memory storage for the collision signals is practically infeasible. Accounting for this fact, the question of the SICTA operation with the single memory location was first addressed in [13]. However, in [13] each data packet was supplied with an extra field/bit with separate feedback, indicating whether the packet is transmitted for the first time. This makes the derived MST incomparable with those for SICTA and conventional TAs as the system model had been changed. Below we present the description of the SICTA modification from [13] and refer to it as to *SICTA with free access* (SICTA/FA) in what follows.

The SICTA/FA protocol (see Fig. 2(e)) extends the possibilities of the free access MTA protocol (see Fig. 2(d)) by supplying it with the SIC property. The additional control field/bit with separate feedback indicates whether a data packet is new (e.g. D in slot 3 of Fig. 2(e)) or a retransmission of the previously collided packet. Having this knowledge, the SICTA/FA may skip a tree level and achieve higher MST. We proceed with the more formal description of the SICTA/FA protocol.

The following PHY-MAC feedback information should be available from the interference cancellation receiver:

1. 'collision' and level skipping (left slot contents extracted) (C/skip);
2. 'collision' and no level skipping (C/-);
3. 'success'/'empty' and no level skipping (SE/-);
4. 'success' and level skipping (left slot contents extracted) (S/skip);
5. 'success'/'empty' and level skipping (deterministic collision in the next slot) (SE/skip).

We denote the captured signal by cs, the stored signal by ss and some meaningful signal by ms. The PHY operation is summarized in Table 1.

Table 1: SICTA/FA PHY operation

Rule	Channel-PHY feedback	Memory	PHY-MAC feedback	Store in memory
1	'Collision'	$ss = cs$	C/skip	cs
2	'Collision'	$ss - cs = ms$	C/skip	cs
3	'Collision'	$cs - ss = ms$	C/skip	ss
4	'Collision'	otherwise	C/-	cs
5	'Success'	$ss - cs = ms$	S/skip	0
6	'Success'	$ss = 0$	SE/-	0
7	'Success', *new**	otherwise	SE/skip	ss
8	'Success', *old*	otherwise	SE/skip	$ss - cs$
9	'Empty'	$ss \neq 0$	SE/skip	ss
10	'Empty'	$ss = 0$	SE/-	0

*Due to the separate feedback it is known whether a data packet is transmitted for the first time (*new*) or retransmitted (*old*).

For each user that participates in a collision resolution process we keep the corresponding level variable L_t at the MAC layer, where t is the slot number. At the initialization stage $L_t = 0$. At the end of slot t each user MAC updates L_t as follows:

1. C/skip: if $L_t \geq 2$, then $L_{t+1} = L_t$
   else if $L_t = 1$, then packet is received successfully
   else $L_{t+1} = \begin{cases} 0, with\ probability\ p \\ 1, with\ probability\ 1-p. \end{cases}$
2. C/-: if $L_t > 0$, then $L_{t+1} = L_t + 1$
   else $L_{t+1} = \begin{cases} 0, with\ probability\ p \\ 1, with\ probability\ 1-p. \end{cases}$
3. SE/-: if $L_t > 0$, then $L_{t+1} = L_t - 1$
   else packet is received successfully.
4. S/skip: if $L_t \geq 2$, then $L_{t+1} = L_t - 2$
   else packet is received successfully.
5. SE/skip: if $L_t \geq 2$, then $L_{t+1} = L_t$
   else if $L_t = 1$, then $L_{t+1} = \begin{cases} 0, with\ probability\ p \\ 1, with\ probability\ 1-p \end{cases}$
   else packet is received successfully.

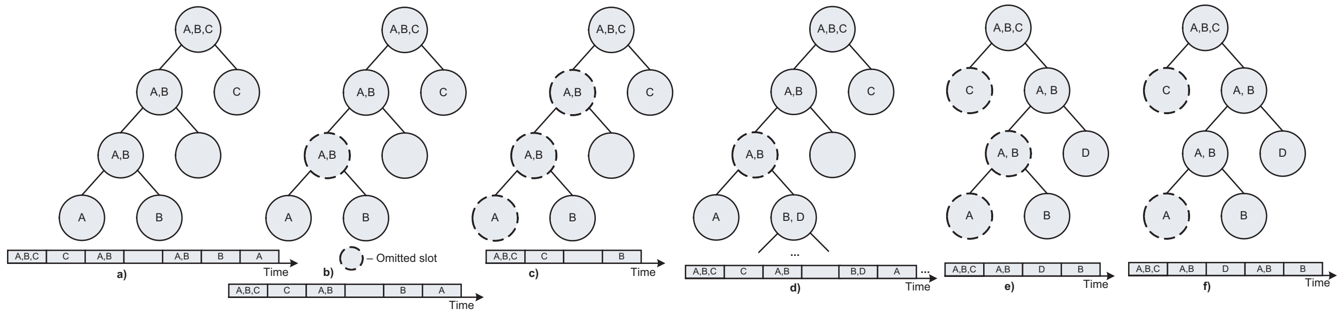

Figure 2: Protocols operation. Gated access: STA a), MTA b), SICTA c). Free access: MTA d), SICTA/FA e), R-SICTA/FA f).

IV. Proposed Protocol

Despite its enhanced performance, the SICTA/FA implementation may be difficult in the practical system. Firstly, it uses an additional bit to provide the necessary feedback information and, therefore, is non-standard in its family. Secondly, it is relatively vulnerable to the imperfect interference cancellation as may force the resolution of a nonexistent collision in case of the cancellation error. This results in a sequence of empty slots unless a newly arrived packet terminates it. In order to make the SICTA/FA protocol both standard and robust we modify the rules of its operation correspondingly. The resulting protocol is termed *robust* SICTA/FA (R-SICTA/FA) and its formal description is as follows.

The following PHY-MAC feedback information should be available from the interference cancellation receiver:

1 - 4. same as for the SICTA/FA protocol (see previous section).

5. 'empty' and level skipping (deterministic collision in the next slot) (E/skip).

We again denote the captured signal by cs, the stored signal by ss and some meaningful signal by ms. The PHY operation is summarized in Table 2.

Table 2: R-SICTA/FA PHY operation

Rule	Channel-PHY feedback	Memory	PHY-MAC feedback	Store in memory
1	'Collision'	$ss = cs$	C/skip	cs
2	'Collision'	$ss - cs = ms$	C/skip	cs
3	'Collision'	$cs - ss = ms$	C/skip	ss
4	'Collision'	otherwise	C/-	cs
5	'Success'	$ss - cs = ms$	S/skip	0
6	'Success'	$otherwise$	SE/-	0
7	'Empty'	$ss \neq 0$	E/skip	ss
8	'Empty'	$ss = 0$	SE/-	0

The MAC operation the R-SICTA/FA protocol is the same as that for the SICTA/FA protocol (see previous section), except for p. 5, where one should formally replace 'SE/skip' feedback with 'E/skip' feedback. Notice that the rules 7 and 8 of the SICTA/FA protocol (see Table 1) are omitted in Table 2 as without the separate feedback it is impossible to distinguish new and old data packets. Furthermore, we emphasize that in case of successful transmission the tree level is not skipped (see Fig. 2(f), slots 3 and 4) except for the situation when a meaningful signal could be extracted after the SIC procedure (see Fig. 2(f), slot 5). Clearly, this modification downgrades the MST of the R-SICTA/FA protocol in comparison to the SICTA/FA for the cost of resistance to cancellation errors.

The proposed protocol may be used with the gated access CAA without any modification and still guarantee resistance to cancellation errors. In case of the gated access scheme the MST of the protocol could be established as the function of the gated STA MST (R_{STA}). Following [16] we notice:

$$R_{STA} \cong \frac{ln2}{2}. \quad (1)$$

We denote the number of the collision resolution tree nodes in a gated STA by n. The number of successful, collision and empty slots during a CRI formed by k collided users is denoted by n_s, n_c and n_e, respectively. As $n_s + n_c + n_e = n$, we use [14] to obtain:

$$n_s = k,\ n_c = \frac{n-1}{2},\ n_e = \frac{n+1}{2} - k. \quad (2)$$

Remember (**Assumption 6**) that after the successful captured signal is canceled from the stored signal the resulting signal contains a meaningless signal with probability q and after the collision captured signal is canceled from the stored signal the resulting signal contains a meaningless signal with probability q' ($q' \geq q$). Below we calculate the expected number of nodes (expected CRI length) in the R-SICTA tree $E[m]$ by subtracting the omitted nodes and accounting for (2). We also account for n_2, that is, the number of collisions of size two during the CRI, to obtain the following:

$$E[m] = (\frac{1}{2} + \frac{1}{4}q')E[n] - \frac{1}{2} + \frac{1}{4}q' + \frac{k}{2} - \frac{1}{2}(q' - q)E[n_2]. \quad (3)$$

Following [16] we establish the bounds on the $\frac{E[n_2]}{k}$:

$$\limsup_{k\to\infty} \frac{E[n_2]}{k} < 0.721355,\ \liminf_{k\to\infty} \frac{E[n_2]}{k} > 0.721340. \quad (4)$$

Using (1) and (4) the final expression for the MST of the gated R-SICTA ($R_{R-SICTA}$) is obtained as follows:

$$R_{R-SICTA} \cong \frac{4}{\frac{2+q'}{R_{STA}} + 2 - 2(q' - q)\frac{E[n_2]}{k}}. \quad (5)$$

V. Numerical Results and Conclusion

In order to quantitatively evaluate the performance of both SICTA/FA and R-SICTA/FA protocols we address two main performance metrics, namely, the MST and the packet delay. In Fig. 3 the MST is plotted as a function of the cancellation error probability q given the successful captured signal. For each SICTA protocol we fix two important special cases of the cancellation error probability q' given the collision captured signal and simulate the MST values.

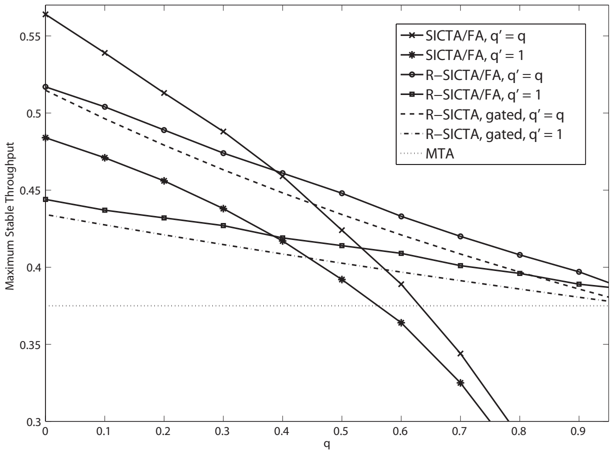

Figure 3: MST comparison.

We observe that the existing SICTA/FA protocol outperforms our proposed R-SICTA/FA protocol when the probability q is relatively low. However, as q increases the SICTA/FA MST degrades rapidly and even drops below the MST of the gated MTA (0.375 for fair splitting). By contrast, the R-SICTA/FA stays higher than the MTA, owing to its resistance to cancellation errors.

We continue with the delay investigation of the SICTA protocols and simulate the mean packet delay (in slots) vs. the arrival rate λ (in packets per slot) in Fig. 4. We compare the SICTA/FA and the R-SICTA/FA protocols for three different values of $q' = q$, namely, 0.1, 0.4 and 0.8. As expected, when $q = 0.1$ the mean delay of the R-SICTA/FA is higher due to its lower MST value (see Fig. 3). In the intermediate case of $q = 0.4$ the mean delays of SICTA/FA and R-SICTA/FA are approximately the same. Finally, when $q = 0.4$ the proposed R-SICTA/FA protocol results in a considerably lower mean delay than its counterpart.

Summarizing, we conclude that despite some performance degradation in comparison to the SICTA/FA protocol, our proposed R-SICTA/FA protocol demonstrates attractive throughput and delay performance, especially when the interference cancellation errors are prevailing.

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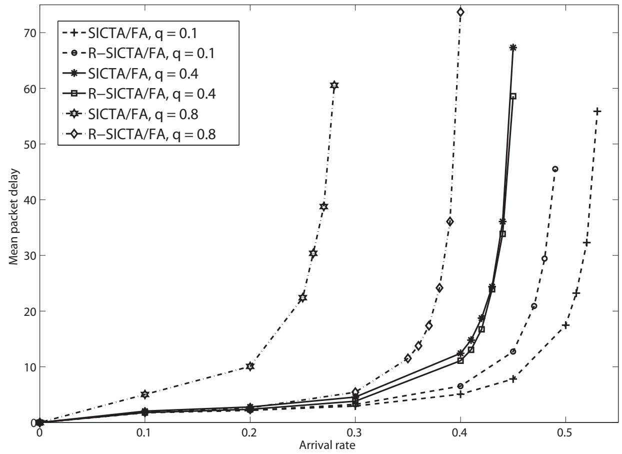

Figure 4: Mean packet delay comparison.

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