# The Choice of the Optimal Segment Size for the Relay with Limited Buffer Resources

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### Abstract

In this paper the Multi-hop ARQ scheme is presented. The question of the additional segmentation in hops is considered. For this scheme it is shown how to choose the optimal segment size in the case of limited and unlimited buffer. Also considered how the delay influences the optimal segment size.

## I. Introduction

Wireless communication and mobile communication technologies have witnessed a fast development in recent years. Among all the other technologies in communication industry, mobile communication, for its wide deployment and easy access nature, has shaped our world in a profound manner. Today, more that one billion people worldwide are mobile subscribers and this number will still be increasing rapidly over the next few years. Now the systems of 3G and 4G are designed for the future needs. But as in the 4G system the higher carrier frequency shrinks the cell coverage of the base station, relay nodes are placed in order to maintain the coverage and quality of service. As was proved in [1] it is more effective to use Multi-hop ARQ in such systems. And even for the easiest case it is necessary to solve the task of the optimal segment size. This paper is structured as follows. The section II introduces the system model. The task of the optimal segment size for the one hope is described in the section III. Section IV presents this task for the case of two hops. The conclusion is presented in the last section.

## **II. System Model**

Let us consider the multi-hop RLC ARQ scheme. To solve the problem of the optimal message segmentation while sending messages from base station to relay node and from relay node to user terminal, it is necessary to be able to solve this problem at least for the way from relay node to user terminal. For solving this task consider several assumptions.

1) Transmitter sends data with the check sum. It is possible to find errors in the message due to this check sum. It is also considered that all errors could be recognized.

2) Receiver send the positive acknowledgement if the message is received successfully and negative acknowledgement if errors occur. It is important to notice that there are no errors in the reverse channel.

3) Transmitter receives acknowledgement after constant period of time  $\tau$  after sending the message. In this paper this delay will be counted in the times of sending one packet.

4) In the forward channel errors occur with the probability of p for one bit of the transmitted message. The probability that an error occurred but was not found equals zero (see assumption 1)

5) All the events connected with error appearance are considered to be independent.

We will consider the case of packets transmission using this set of assumptions and answer the question of packets additional segmentation.

III. Single hop case

Relay node should be designed as low-cost low transmit power devices compared to the BS, so the buffer size should be limited.

The throughput depends on the following values:

- Bit error probability
- Packet size
- Number of segments in the packet
- Length of check sum
- Delay

• Type of used algorithm and number of segments that could be stored on the receiver side.

It is necessary to underline that although we transmit the packet with the check sum, there is no need to store this check sum at the receiver side.

In general the formula for channel throughput is calculated in two steps:

$$\eta = AB$$
.

The first multiplier is the following coefficient:

$$\frac{k/N}{k/N+r},$$

where k - length of the initial packet, N - number of segments in the packet, r - length of the check sum.

Second multiplier takes into account the particular qualities of the ARQ algorithm. For its calculation the delay value y (computed depending on the size of the packet) and probability of the segment error q:

$$y = \left[\frac{\tau(k+r)}{k/N+r}\right],$$
$$q = 1 - (1-p)^{k/N+r},$$

where  $\tau$  - initial delay value in the channel, k - initial packet size, N - number of segments in the packet, r - length of the check sum, p - probability of the bit error.

For computing the value of probability of segment error, we take an assumption that the probability of bit error does not depend on the segment size.

For Go Back N algorithm second multiplier equals

$$\frac{1-q}{1+yq}.$$

For selective repeat algorithm when we store one segment at the receiver side the second multiplier is computed as following [2]:

$$\frac{1}{1 + \frac{q(y+1)(1+q)}{1+q-2q^2}}.$$

For selective repeat algorithm when more than one segment is stored at the receiver side this multiplier is estimated by the simulation.

If there is no buffer constraint at the receiver side the following formula is used:

1 - q.

Formulas for Go Back N algorithm and selective repeat with infinite buffer can be found in many papers, for example [3].

These computations were made based on the assumption that bit error probability does not depend on the segment size. In practice, bit probability error decrease with the increase of the segment size. However, even without this feature the computations show that there is no need for additional segmentation on the relay node when there is buffer constraint.

Graphs (Figure 1) was built with the parameters defined in Table 1. It is clear from the Figure 1 that for high probability values there is a small gain of the algorithm with limited buffer with segmentation when either the Go Back N algorithm is used or selective repeat algorithm. It also

shows that in the case of limited buffer on the receiver side segmentation is not needed and with the increase of the probability the difference becomes significant. So, the segmentation is needed only for high error probability value or in case of the unlimited buffer.



Fig. 1. Different ARQ schemes comparison

## IV. Two hops case

#### A. Selective repeat algorithm with unlimited buffer

Consider the case of the selective repeat algorithm with infinite buffer on the relay. Consider the same assumptions as in section IV. Data is transmitted from BS through RN to UT. It is also considered that successfully received packets on the UT are immediately sent to the upper layers. The model is presented on the Figure 3. Let  $p_1$  be the probability bit error on the hop from BS to Relay, and  $p_2$ - probability bit error on the hop from Relay to UT. With the above-mentioned assumptions for transmitting one segment on the first and second hops accordingly we need

$$T_{serv.1} = \frac{1}{1 - (1 - p_1)^{\frac{k}{n} + r}}$$

and

$$T_{serv.2} = \frac{1}{1 - (1 - p_2)^{\frac{k}{n} + r}}$$

time slots. One time slot is the time of transmitting one segment.

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Consider two cases.

1) Let  $SNR_1 \le SNR_2$ , therefore  $p_1 > p_2$ , then  $\overline{T}_{serv.1} > \overline{T}_{serv.2}$ .

The work of the RN can be presented as the queuing system with one server with service time  $T_{serv.2}$  with the arrival rate of  $\lambda_2 = 1/T_{serv.1}$ . Then following inequality holds true  $\lambda_2 \cdot T_{serv.2} = \frac{T_{serv.2}}{T_{serv.1}} < 1$ .

If  $p_1 > p_2$ , then for any length of the segment on the relay node will be finite queue, so we can choose the segment length so that optimize the throughput on the first hop.

2) Let  $SNR_1 > SNR_2$ , therefore  $p_1 < p_2$ , then  $\overline{T}_{serv.1} < \overline{T}_{serv.2}$ .  $\lambda_2 \cdot T_{serv.2} = \frac{T_{serv.2}}{T_{serv.1}} > 1$ , therefore the

system will work unstable and the queue will be overfilled. So the relay has to send negative acknowledgement and the work of the system will be defined by the worst hop.

So the optimal number of segments will be chosen as following:

$$\frac{k/n}{k/n+r} (1 - \max(p_1, p_2))^{k/n+r}.$$

The optimal value of n will be the value that maximize the above-stated formula.

## B. Go Back N algorithm with limited buffer

For the above-mentioned case with the infinite buffer, the delay does not influence the throughput for the selective repeat algorithm. Let us now generalize these arguments for the case of limited buffer. Let RN and UT have limited buffer. Go Back N algorithm is used for the transmission.

The delay between the moment of the end of message transmission by BS and the moment of receiving the acknowledgment from RN equals  $\tau_1$  (i.e. it is possible to transmit  $\tau_1$  packets during this period of time). The delay between the moment of the end of message transmission by RN and the moment of receiving the acknowledgment from UT equals  $\tau_2$ . For simplicity we limit our discussion to the case when SNR on the hop between BS and RN is bigger than between RN and UT, (i.e.  $p_1 < p_2$ ). Taking into account all assumptions the time from message sending on the first hop equals

$$T_{serv.1} = \frac{1 + y_1 q_1}{1 - q_1}$$
 and on the second  $T_{serv.1} = \frac{1 + y_2 q_2}{1 - q_2}$ 

Consider three cases.

1)  $\tau_1 = \tau_2$ . Let prove that for equals  $\tau_1$  and  $\tau_2$  everything will be determined by the second hop.

$$T_{serv.1} = \frac{1+y_1q_1}{1-q_1} = \frac{1+\left|\frac{\tau_1(k+r)}{k/N+r}\right| \left(1-(1-p_1)^{k/N+r}\right)}{\left(1-p_1\right)^{k/N+r}} = \frac{1}{(1-p_1)^{k/N+r}} + \frac{\left|\frac{\tau_1(k+r)}{k/N+r}\right|}{(1-p_1)^{k/N+r}} - \left\lceil\frac{\tau(k+r)}{k/N+r}\right\rceil}{T_{serv.2}} = \frac{1}{(1-p_2)^{k/N+r}} + \frac{\left\lceil\frac{\tau_2(k+r)}{k/N+r}\right\rceil}{(1-p_2)^{k/N+r}} - \left\lceil\frac{\tau_2(k+r)}{k/N+r}\right\rceil}$$

Let  $T_{serv.1} < T_{serv.2}$ , then

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$$\frac{1}{\left(1-p_{1}\right)^{k/N+r}} + \frac{\left\lceil \frac{\tau_{1}(k+r)}{k/N+r} \right\rceil}{\left(1-p_{1}\right)^{k/N+r}} - \left\lceil \frac{\tau_{1}(k+r)}{k/N+r} \right\rceil < \frac{1}{\left(1-p_{2}\right)^{k/N+r}} + \frac{\left\lceil \frac{\tau_{2}(k+r)}{k/N+r} \right\rceil}{\left(1-p_{2}\right)^{k/N+r}} - \left\lceil \frac{\tau_{2}(k+r)}{k/N+r} \right\rceil$$

As  $\tau_1 = \tau_2$  then

$$\frac{1}{\left(1-p_{1}\right)^{k/N+r}} + \frac{\left\lceil \frac{\tau(k+r)}{k/N+r} \right\rceil}{\left(1-p_{1}\right)^{k/N+r}} < \frac{1}{\left(1-p_{2}\right)^{k/N+r}} + \frac{\left\lceil \frac{\tau(k+r)}{k/N+r} \right\rceil}{\left(1-p_{2}\right)^{k/N+r}}$$

As  $p_1 < p_2$ , then this statement holds true, therefore, the assumption  $T_{serv.1} < T_{serv.2}$  also holds true, then everything will be determined by the second hop.

2)If  $\tau_1 < \tau_2$ , then as in case one everything is determined by the worst throughput, as in this case  $T_{serv.1} < T_{serv.2}$ 

3) If  $\tau_1 > \tau_2$ , then for some values  $T_1$  can be more than  $T_2$ . Therefore, we should higher SNR level. Let us find out the values of  $p_1$ ,  $p_2$ ,  $\tau_1$ ,  $\tau_2$  when this could be possible.

Consider the following graphs (Figure 2 and Figure 3). The lower line corresponds to the case of the infinite buffer. The area under it is not interesting for us as we defined that  $p_1 < p_2$ . The area between these two lines corresponds to the situation when the optimal segment size is determined by the first hop. The more the difference between  $\tau_1$  and  $\tau_2$ , the bigger this area is. The area higher than line with circles is the area where optimal segment size is determined by second hop.



Fig.2.  $\tau_1 = 2\tau_2$ 



C. Generalization for the case of the buffer of arbitrary size

The discussion stated above could be generalized for the case when UT and RN have the unlimited buffer and the selective repeat algorithm. In fact, two extreme cases are considered. In the case of the infinite buffer the area when the segment size will be determined by the hop with the highest SNR is absent. With the buffer limiting this area extends and the border of this area is the line that corresponds to the above-mentioned case of Go Back N algorithms. It is possible to determine such an area by combining numerical computations and simulation for the buffer of finite size.

### V. Conclusion

For the single hop case it is shown that for the case of the limited buffer the segmentation is unnecessary. If the case of the two concatenated hops is observed, then for the situation of unlimited buffer the segment size should be chosen according to the worst hop. For the case of the limited buffer it might occur that everything will be determined by the hop with the higher delay value despite it might have had less probability bit error.

#### References

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