THE MULITPLE-RANDOM-ACCESS ALGORTHMS ANALYSIS BASED ON TREE PROPERTIES

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Abstract. The idealized model and different-slot-duration model of Random-Mulitple-Access noisy Channel are considered. The relations between rates for these models, obtained by method based on simple properties of trees are presented.

Introduction.

At first we consider the idealized model of Random-Mulitple-Access (RMA) system [1]. We use the following notations for the description of the collision resolution tree in the case of blocked non-modified algorithms: l - the branching coefficient, k - the number of conflicting users, n_k - the total number of the vertices in the tree, $n_k^{(e)}$, $n_k^{(s)}$ и $n_k^{(c)}$ - the number of the vertices, corresponding to the "empty" slots, to the "success" slots and to the "collision" slots respectively.

It is easy to prove, that for the considered tree it is true, that

$$n_k^{(e)}+ n_k^{(s)}=(l-1)n_k^{(c)}+1, \qquad (1)$$

$$n_k^{(e)}+ n_k^{(s)}+ n_k^{(c)}=n_k, \qquad (2)$$

$$n_k^{(s)}=k, \qquad (3)$$

which results in

$$n_k^{(c)}=(n_k-1)/l, \qquad (4)$$

$$n_k^{(e)}=(l-1)(n_k-1)/l + 1 - k . \qquad (5)$$

Suppose that the lower and upper bounds for the rate R_n of the blocked-non-modified collision resolution algorithm are known. In what follows we use these bounds together with relatons (1-5) in order to obtain the rate of modified algorithm for the idealized model, the rate of non-modified algorithm for the false-conflict model, and the rates of modified and non-modified algorithms for the different-slot-duration model. Our considerations are also based on the following formula:

$$\underline{\lim}_{k \to \infty} \frac{k}{T_k} < R < \overline{\lim}_{k \to \infty} \frac{k}{T_k} , \qquad (6)$$

where T_k is the average resolution time for k-collision.

For the sake of simplification we also suppose $l=2$.

The rate of the modified algorithm.

The collision resolution tree for the considered case is described by following values:

m_k - the total number of the vertices in the tree,

$m_k^{(e)}$, $m_k^{(s)}$и $m_k^{(c)}$- the number of the vertices, corresponding to the "empty" slots, to the "success" slots and to the "collision" slots respectively.

It is true that

$$m_k^{(s)} = n_k^{(s)} = k, \qquad (7)$$

$$\mathbf{E}[m_k^{(e)}] = \mathbf{E}[\ n_k^{(e)}], \qquad (8)$$

$$\mathbf{E}[m_k^{(c)}] = \mathbf{E}[n_k^{(c)}] - \mathbf{E}[n_k^{(e)}]/2, \qquad (9)$$

where $\mathbf{E}[x]$ is the mean value of random variable x.

Using (9) together with (1-5) we obtaine the relation between the numbers of the vertices in the trees for modified and non-modified algorithms:

$$\mathbf{E}[m_k] = (3\ \mathbf{E}[n_k\ ] - 1)/4 + k/2\ . \qquad (10)$$

Denote by Rn and Rm the rates of non-modified and modified algorithms. Using (6) together with (10) we see, that

$$R_m \approx (3\ /\ (4\ R_n) + 1/2\ )^{-1}. \qquad (11)$$

Calculation of the rate for the noisy channel.

Let q_e, q_s be the probabilities of false collision detection in the "empty" and "success" slots respectively [2]. Let also false collisions be independent for the different slots. One can show that the values in (1)-(3) depend on q_e, q_s in the following way [3]:

$$n_k^{(c)}(q_e,q_s) = n_k^{(c)}(0,0), \qquad (12)$$

$$n_k^{(e)}(q_e,q_s) = n_k^{(e)}(0,0) \;\; n_0\,(q_e,q_s), \qquad (13)$$

$$n_k^{(s)}(q_e,q_s) = n_k^{(e)}(0,0) \;\; n_1\,(q_e,q_s), \qquad (14)$$

where $n_k^{(\cdot)}(q_e,q_s)$ is the number of the vertices of the tree for the given values q_e, q_s.

Summing up equations (12)-(14) with $q_e=q_s=q\leqslant1/2$ we obtain

$$T_k = E[n_k\ (q_e,q_s)]=E[n_k\ (0,0)](1-q)/(1-2q)+q/(1-2q), \quad (15)$$

which results (using (6)) in

$$R(q_e,q_s)\approx R(0,0)(1-2q)/(1-q).$$

One can prove that for $l>2$ it holds that

$$R(q_e,q_s)\approx R(0,0)(1-l\cdot q)/(1-q).$$

Calculation of the rate for different-slot-duration channel.

Let τ_e,τ_s,τ_c be the durations of the "empty", "success" and "collision" slots respectively [4]. Then the average resolution time T_k for k-collision is given by

$$T_k=E[\tau_e\cdot n_k^{(e)}+\tau_s\cdot n_k^{(s)}+\tau_c\cdot n_k^{(c)}], \qquad (16)$$

which results in

$$R_n(\tau_e,\tau_s,\tau_c) \approx ((\tau_e+\tau_c)/(2R_n(1,1,1))+(\tau_s-\tau_e))^{-1}, \qquad (17)$$

where $R_n(\tau_e,\tau_s,\tau_c)$ is the rate of the non-modified algorithm with given values of τ_e,τ_s and τ_c.

The following expression for the rate $R_m(\tau_e,\tau_s,\tau_c)$ of the modified algorithm is derived in similar manner

$$R_m(\tau_e,\tau_s,\tau_c) \approx ((\tau_e/2+\tau_c)/(2R_n(1,1,1))+(\tau_s-\tau_e+\tau_c/2))^{-1}.$$

Generalized result for the dynamic and part-and-try algorithms.

Denote the rates of the dynamic and part-and-try algorithms by $R_{dyn}(\tau,\tau_s)$ and $R_{part}(\tau,\tau_s)$ respectively for the case of $\tau = \tau_e=\tau_c$. It can be proved that

$$R_{dyn}(\tau,\tau_s) = (\ \tau\ /R_{dyn}(1,1)+(\tau_s-\tau))^{-1},$$
$$R_{part}(\tau,\tau_s) = (\ \tau\ /R_{part}(1,1)+(\tau_s-\tau))^{-1}.$$

Conclusion.

The presented approach simplifies the analysis of tree algorithms for the various channel models. The values of interest for these models are obtained from the same values for the *idealized model in explicit form.*

References

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